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Adjoint Method Applied in Optimization of the Overhead Transmission Lines Configuration

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Abstract:
In this paper, a new methodology is applied for the optimization process of the geometry of the bundle of conductors of the overhead transmission lines. This methodology is based on the application of the Adjoint Method to calculate the sensitivity of the parameters related to the charge electric of the transmission lines. This information is used with a Gradient Method and a Gold Section algorithm for the minimize the electric field at ground level. The Surge Impedance Loading is calculated after the optimization process for check improvements in energy capacity transportation.

Keywords: Transmission lines; Adjoint Method; Sensitivity Analysis; Electric Charge; Gradient Method.

1. INTRODUCTION

Brazil is a country of large territorial extension. The large extent of its territory denotes the need for very long transmission lines (TL) which are very expensive. The increasing demand for electrical energy makes mandatory the implementation of new efficient methods of power transmission. Therefore, it is necessary to develop new studies aimed to increase the transmission lines capacity, Paganotti et al. (2015).

This increase in the TL capacity can be achieved through some techniques such as: increasing the thermal limit of the line, Hall and Deb (1988), EPRI (2005); increasing the operating voltage or the number of conductors per phase or using compensators, SALARI and ESTRELLA JR (2019), Gomes Jr et al. (1995), Sarmiento and Tavares (2016).

The High Surge Impedance Loading Transmission Lines (HSIL) is one non-conventional alternative that is highly viable, Acosta and Tavares (2018). This technology comprises the rearrangement or the increase of the conductor's number per phase to equalize the electric field between them. The HSIL implementation has as fundamentals the understanding of the electric field behavior associated with each conductor and how the physical and geometrical parameters of the transmission line affect the field distribution, Melo et al. (1999a), Alexandrov (1969).

Determine efficiently the sensitivity of the electric fields and capacity of energy transportation of the transmission lines has been studied by researchers and energy companies, Acosta and Tavares (2018).

This paper applied a sensitivity analysis based on the Adjoint Method, Bakr et al. (2017). This methodology has been used with success on forecasting wheater studies, electric engineering problems and in problems with elevate dimension, Bakr (2013), Dadash et al. (2012), Nikolova et al. (2004). This method gives the sensitivity analysis of a problem with n variables solving only one more system of equations. While on classical approximation using difference finite is necessary to solve one system of equations by each variable, Bakr (2013). The Adjoint method is a very efficient and fast way to get gradient information for using optimization problems. Gradient method, Ellipsoidal methods, Quasi-Newton Methods, and others can be used with Adjoint modeling, Luenberger et al. (1984), (Paganotti et al., 2015).

The sensitivity analysis of the parameters of low-frequency electromagnetic problems using Adjoint Modeling is not currently in the literature. This kind of evaluation is based on Telegen's Theorem and have been employed on high-frequency problems and on the time domain for electric circuits analysis, Director and Rohrer (1969).

This assignment aims to develop a computational tool to calculate and minimize the levels of the electric field. An electromagnetic model of TL understudy is developed and the charge and the electric field intensity of the system are calculated. The optimization procedure is performed by a Gradient Method using the sensitivity information from Adjoint Modelling of the problem. The High Surge Impedance Loading is calculated after the optimization process.

This paper is organized as follows. Section 2 modeling of transmission lines to calculate the electric field. In Section 3 the adjoint modeling is developed. Section 4, a brief introduction to the optimization method is given. Also, the purposed fitness functions are illustrated in this section, and it is the application in the electric field at the ground level is discussed. In Section 5 the surge impedance loading of transmission lines is discussed. In Section 6, simulation results are presented. The acknowledgments are given in the Section 7. Section 8 is the conclusion.

2. MODDELING OF TRANSMISSION LINES

The TL's operating regime adopted is quasi-static. The domains that involve the problem, are considered linear, homogeneous and isotropic, Balanis (1998). For the electric field calculation, the transmission line's conductors are modeled as parallel cylindrical straight conductors to the ground. The soil's effect is taken into account using the Method of Images. The electric charge of each phase is obtained by Maxwell coefficients matrix, Balanis (1998):

$$\begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} = \begin{pmatrix} P_{aa} & P_{ab} & P_{ac} \\ P_{ba} & P_{bb} & P_{bc} \\ P_{ca} & P_{cb} & P_{cc} \end{pmatrix}^{-1} \cdot \begin{pmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{pmatrix} \tag{1}$$

In (1) q_a, q_b and q_c are the electric charge phasors of each phase, V_{an}, V_{bn} and V_{cn} are the voltage phasors applied in each phase. The elements of the Maxwell Potential coefficients matrix are given by, Balanis (1998):

$$P_{km} = \frac{1}{2\pi\epsilon_0} \ln\left(\frac{H_{km}}{D_{km}}\right) \tag{2}$$

In (2), in case of $k = m$, H_{km} is the distance between each conductor and its image and D_{km} is the radius of the conductor; in case of $k \neq m$, H_{km} is the distance between the image conductor k and m and D_{km} is the distance between the conductors k and m. The constant ϵ_0 is the electric permitivity of vacuum, Balanis (1998).

Once the electric charge of each phase determined, the electric field in x and y directions at the ground level can be found applying the Gauss's law, Balanis (1998):

$$\boldsymbol{E}(x, y) = E_x \hat{a}_x + E_y \hat{a}_y \tag{3}$$

Where E_x is given by, Balanis (1998):

$$E_x(x, y) = \sum_{i=1}^{N} \left(\frac{q_i}{2\pi\epsilon_0}\right) \times \left[\frac{(x_n - x_i)}{(x_n - x_i)^2 + (y_n - y_i)^2} - \frac{(x_n - x_i)}{(x_n - x_i)^2 + (y_n + y_i)^2}\right]^2 \tag{4}$$

The electric field at ground level on x direction can be obtained employing (4). Where, q_i is the electric charge of the i_{th} conductor; x_i and y_i, x_n and y_n, are respectively the horizontal and vertical positions of the source conductors and the point of field evaluation; and N is the number of the conductors. The electric field on y direction is similarly obtained. Note that, the electric field calculation depends on the conductor's positions and the charge of each conductor. The position of each conductor affects the capacitance and inductance parameters, these are decisive in the TL capacity, Glover et al. (2012).

The analysis of the sensitivity of the charge electric of the system obtained by (1) related to spatial coordinates (x, y) is obtained by the Adjoint Method in the next section.

3. ADJOINT MODELLING

The charge electric of each conductor of the transmission line is obtained by a linear system of equations given by:

$$P(x, y)q = V \tag{5}$$

In (5), $P(x, y)$ is the matrix of the system with dimension $\{N\ x\ N\}$, where N is the number of conductors. The elements of this matrix are dependents of the x and y coordinates. q is the vector of state variables and V is the excitation vector, the both have dimensions $\{N\ x\ 1\}$.

The aim of the Adjoint Method is to find the gradient of a response function defined by the user $f(x, y, q)$ with respect x and y. This response generally is the objective function of a optimization problem. The classic way to do it, is to perturb each parameter with respect x_i and y_i and solve the linear system obtained with each perturbed parameter.

The finite difference is the approximation commonly used to determine the sensitivity this way. This approximation needs to build $P(x, y)$ for each perturbed parameter and solve the linear system at least n times, Bakr (2013).

The Adjoint Method can estimate the required sensitivity on way more efficiently. The analysis of sensitivity related on x coordinate is explained in this section. Initially, the linear system given by (5) is differentiate in relation of i_{th} parameter x_i, Bakr et al. (2017):

$$\frac{\partial(P(x)\bar{q})}{\partial x_i} + P\frac{\partial q}{\partial x_i} = \frac{\partial V}{\partial x_i} \tag{6}$$

The first term of (6) consists of differentiating P matrix while q is maintained unchanged in your nominal values $\bar{q}$. Solving for the derivate of state variables is obtained:

$$\frac{\partial q}{\partial x_i} = P^{-1}\left(\frac{\partial V}{\partial x_i} - \frac{\partial(P\bar{q})}{\partial x_i}\right) \tag{7}$$

The derivate of response function of interest $f(x,q)$ in relation of i_{th} parameter x_i is given by, Bakr et al. (2017):

$$\frac{\partial f}{\partial x_i} = \frac{\partial^e f}{\partial x_i} + \left(\frac{\partial f}{\partial q}\right)^T P^{-1} \left(\frac{\partial V}{\partial x_i} - \frac{\partial (P\bar{q})}{\partial x_i}\right) \tag{8}$$

In (8) the elements dependents of x parameters are used to define the Adjoint estate variable using, Bakr et al. (2017):

$$\hat{q}^T = \left(\frac{\partial f}{\partial q}\right)^T P^{-1} \tag{9}$$

$$\hat{q} = (P^T)^{-1} \left(\frac{\partial f}{\partial q}\right) \tag{10}$$

$$P^T \hat{q} = \left(\frac{\partial f}{\partial q}\right) \tag{11}$$

The vector $\hat{q}$ of Adjoint variables is obtained solving (11). The matrix Adjoint of the system is the transpose matrix of the original system given by (5). The excitation of the Adjoint System (11) depends of response function $f(x,q)$ and your derivates related with the state variables. Solving the Adjoint system the sensitivity of the response for the i_{th} parameter V_i is given by, Bakr et al. (2017):

$$\frac{\partial f}{\partial x_i} = \frac{\partial^e f}{\partial x_i} + \hat{q}^T \left(\frac{\partial V}{\partial q} - \frac{\partial (P\bar{q})}{\partial x_i}\right) \tag{12}$$

Once time that solution of the original system (5) gives q and solving adjoint system (11) where $\hat{q}$ is obtained, the sensitivity related a every x parameter can be obtained using (12).

In the problem of charge electric of transmission lines treated in this paper the phasorial vector of excitation V is known. In this case, V has no dependence on x and the derivate related disappeared. The response function $f(x,q)$ adopted in this paper is given by:

$$f(x,q) = \left(\sum_{i=1}^{N_{cond}} q_i\right)^2 \tag{13}$$

The derivate of response function adopted as excitation vector in the Adjoint model is:

$$\frac{\partial f(x,q)}{\partial q} = 2\left(\sum_{i=1}^{N_{cond}} q_i\right) \tag{14}$$

The response function have not a explicit dependence on x then in (12) the first right term is null. The sensitivity analysis expression obtained with the adjoint model is finally obtained:

$$\frac{\partial f(x,q)}{\partial x_i} = \hat{q}^T \left(-\frac{\partial P}{\partial x_i}\right) \bar{q} \tag{15}$$

Similarly, on y direction obtain:

$$\frac{\partial f(y,q)}{\partial y_i} = \hat{q}^T \left(-\frac{\partial P}{\partial y_i}\right) \bar{q} \tag{16}$$

In (15) and (16) q is obtained by solution of the Adjoint model and q is obtained by the solution of the original system. With the Adjoint method is possible to get the information of Sensitivity Analysis of n variables solving only more one linear system. If to use classical approximation like finite difference the Sensitivity Analysis envolves to solve and inverse n linear systems.

The derivate of the matrix of system P is obtained using central finite difference aproximattion. The i_{th} derivate of each element of P is, Bakr (2013):

$$\frac{\partial P}{\partial x_i} = \left(\frac{P(x_i + \Delta x_i) - P(x_i - \Delta x_i)}{2\Delta x_i}\right) \tag{17}$$

Once time defined the way that the Gradient information is obtained the next step is to choose an optimization method. In the future works, the Finite Element Method(FEM) modeling of the transmission lines will be developed for using the Adjoint Method to calculate the sensitivity analysis of parameters related to the electric field and SIL of the transmission lines.

4. OPTIMIZATION METHOD

In order to optimize the aforementioned TL model, deterministic methods have been used, Maciel et al. (2013), Lalau-Keraly et al. (2013). Using the Adjoint modeling, developed in the section before, to calculate the gradient of the interest function, one method of optimization that needs this information is adopted. The Gradient method with a Gold Section Method is used. The method is defined by simplified iterative algorithm, Bakr (2013):

$$x_{k+1} = x_k - \alpha \nabla f(x) \tag{18}$$

In (18) from the point x_k is search along the direction of the negative gradient to a minimum point on this line, this minimum point is taken to be x_{k+1}. α is a non-negative scalar minimizing term it's changing in each iteration and is calculated by Golden Section Algorithm, Bakr (2013). The unique condition necessary for application this algorithm is that the function must be differentiable. The complete algorithm description is given by Luenberger et al. (1984). The square of the sum of the intensity of charge electric by each conductor of the TL obtained by (1) is the objective function for this work:

$$f(x,q) = \left(\sum_{i=1}^{N_{cond}} q_i\right)^2 \tag{19}$$

In (19) N_{cond} is the number of conductors of the transmission line. In the optimization process, the conductor's height can vary position to a maximum and minimum value which is set as 0.50 meter above or below of the original positions. Figure 1 show one diagram illustrating the restrictions adopted in the optimization process.

Figure 1 specify D_{min} which determines the minimum distance between different phases and d_{min}, which de-

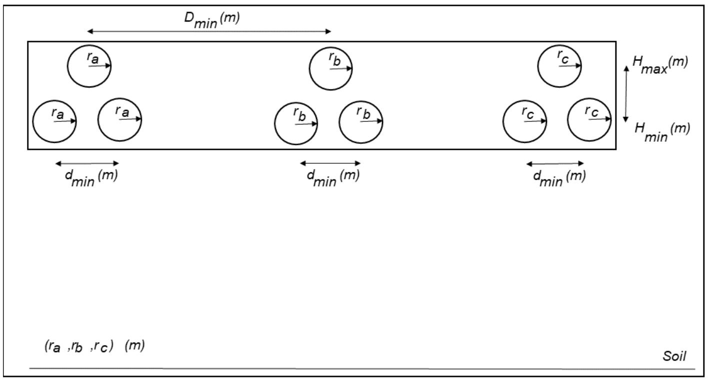

Figure 1. Geometric Constraints

termines the minimum distance between conductors of a given phase.

To simplify the optimization process, the shield wires are disregarded, since their effect in the electric evaluation at ground level is negligible, Sarma and Janischewskyj (1969). After the application of the optimization process, the Surge Impedance Loading (SIL) of the original and optimized configuration is obtained.

5. SURGE IMPEDANCE LOADING OF TRANSMISSION LINES

A TL with enhanced capacity of energy transportation can be achieved by the improvement of Surge Impedance Loading (SIL). The SIL is the MW loading of extra high voltage transmission lines (EHV) at which natural reactive power balance occurs. Reactive power is produced by the line depending on its capacitance and voltage level and consumed in the line to support magnetic field. Balance of both consumption and production of reactive power by line at particular level results into flat voltage profile along the line and keeps the angular and voltage stability within limits, Kiessling et al. (2014).

The Surge Impedance($\mathrm{Z_s}$) is mathematically expressed as reactive power produced being equal the reactive power consumed. This relationship can be expressed as, Kiessling et al. (2014):

$$Z_s = \frac{V_{ff}}{I} = \sqrt{\left(\frac{L}{C}\right)} \tag{20}$$

In (20), V_{ff} is the voltage between two phases, I is the line current $[A]$, L is the line inductance of positive sequence by meter $[H/m]$, C is the line capacitance of positive sequence by meter$[C/m]$ and Z_s is the Surge Impedance $[Ohms]$. The SIL can be expressed as, Kiessling et al. (2014):

$$SIL = \frac{{V_{ff}}^2}{Z_s} \tag{21}$$

From (21) and (20) it is seen to increase the Surge Impedance Loading level, line inductance (L) is to be reduced and/or capacitance(C) is to be increased. The High Surge Impedance Loading Transmission Lines($HSIL$) adopted reduced distance between phases and improved distance among the subconductors. Other procedures adopted are the augmented number of the conductors by phase and the non-conventional configuration in each phase, Melo et al. (1999b).

The sensitivity analysis of the positions of the cables using adjoint modeling related to the electric field at ground level is a new approach proposed in this work. The configuration suggested by the optimization process must be analyzed by their technical feasibility of construction, through studies of mechanical stress, costs, wind efforts among others, Régis Jr et al. (2009).

6. RESULTS

A 345 kV and 500 kV TL configuration with two and three conductors by phase are considered, respectively. The constraints adopted are shown in Table 1, where D_{min} is the minimum distance between different phases, d_{min} is the minimum distance between conductors of each phase, X_e and X_d are the horizontal limits, and H_{min} and H_{max} are the vertical limits.

Table 1. Constraints of the Optimization Process

Case (m)	D_{min} (m)	d_{min} (m)	X_e (m)	X_d (m)	H_{max} (m)	H_{min} (m)
I	5.00	0.40	-9.45	9.45	14.79	13.79
II	6.00	0.45	-12.23	12.23	15.97-15.57	14.77-14.38

In the two cases, the gradient information is obtained in two ways. First using the Adjoint method developed in Section 3 and second is calculated using the Central Finite Difference (CFD) approximation. The error between both ways to calculate the sensitivity is presented in Table 2.

Table 2. Comparative Error - Adjoint Method and CFD Approximation

Case I	Gradient in x	Gradient in y
Cable 1 Adjoint Method	1.091805838e-12	3.181985954e-14
Cable 1 CFD Method	1.091805831e-12	3.181985975e-14
Erro (%)	-6.611686784e-09	6.712218848e-09
Case II	Gradient in x	Gradient in y
Cable 1 Adjoint Method	2.029911575e-12	4.681604714e-13
Cable 1 CFD Method	2.029911565e-12	4.681604738e-13
Erro (%)	-4.686495184e-9	5.215342565e-9

In Table 2 the conductor 1 of both configurations are chosen for the error analysis. It can be noted that sensitivity on x and y directions obtained by the Adjoint Method have high precision with error only next sixth decimal case. This fact shows that adjoint modeling can give gradient information with elevate precision, faster and efficiently of that finite-difference approximation.

For the Case I, TL 345 kV, two conductors by phase, the numbers of iterations of the method are 300. The profile of the original and the optimized electric field can be seen in Figure 2.

The Figure 2 shows that the application of the Gradient Method with the Adjoint Method gives a new configuration with a significant reduction on the electric at ground level. The original and optimized configuration of conductors can be observed in Figure 3.

In Figure 3 the configuration is symmetric the algorithm reduces the distance between the different phases and

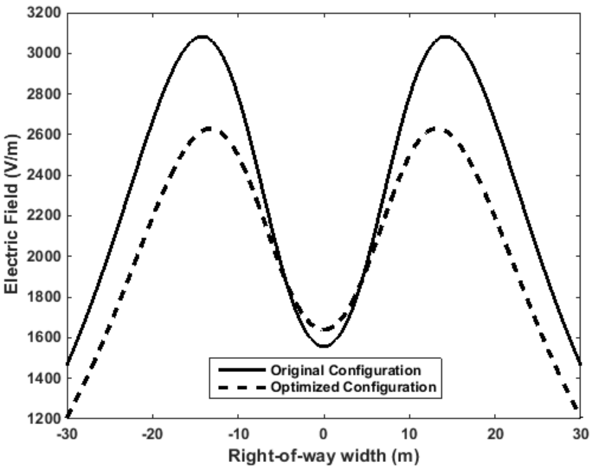

Figure 2. Original and Optimized Profiles of Electric Fields - Case I - Two Conductors

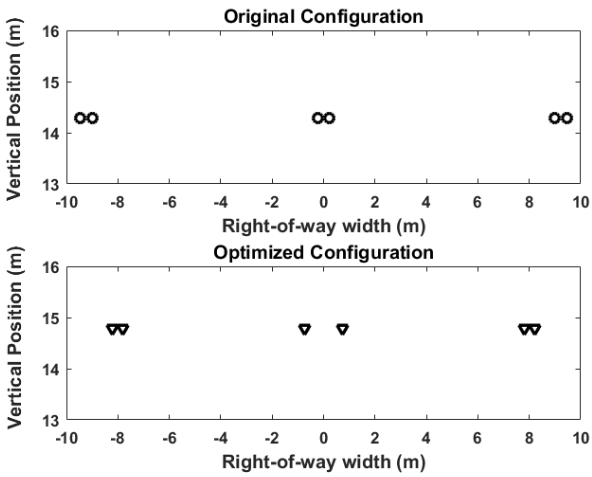

Figure 3. Original and Optimized Configuration - Case I - Two conductors

increases the height of the conductors. The initial and final positions of conductors are shown in Table 3.

Table 3. Original and Optimized Positions

Case I	Phase 1 (m)	Phase 2 (m)	Phase 3 (m)
x_{orig}	-9.45 -9.00	-0.22 0.22	9.00 9.45
x_{otim}	-8.22 -7.82	-0.75 0.75	7.82 8.22
y_{orig}	14.29 14.29	14.29 14.29	14.29 14.29
y_{otim}	14.79 14.79	14.79 14.79	14.79 14.79

The Surge Impedance Loading (SIL) of the original and optimized configuration is given in Table 4. The SIL of the new configuration is 6.35 higher than the original one.

Table 4. Case I - Surge Impedance Loading(SIL) and Surge Impedance(Z_c)

Case I	Original	Optimized
$Z_c(Ohms)$	294.40	276.83
$SIL(MW)$	404.29	429.95
Increase SIL (%)	6.35	

For the Case II, TL 500 kV, three conductors by phase, the number of iterations adopted by the Gradient method are 300. The profile of the original and the optimized electric field can be observed in Figure 4.

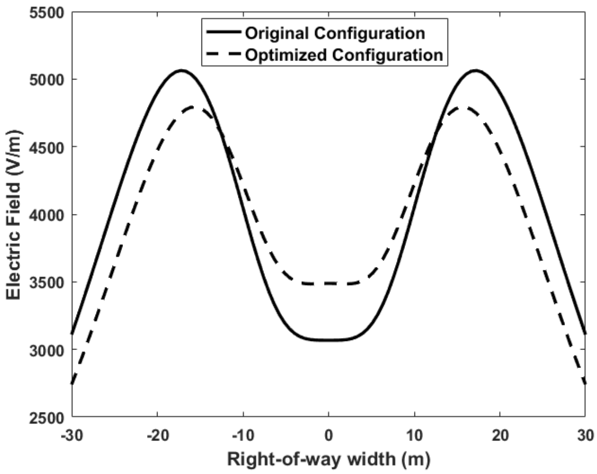

Figure 4. Original and Optimized Profiles of Electric Fields - Case II - Three Conductors

The Figure 4 shows that the application of the Gradient Method with the Adjoint Method gives a new configuration with a reduction on the electric at ground level. The original and optimized configuration of conductors can be observed in Figure 5.

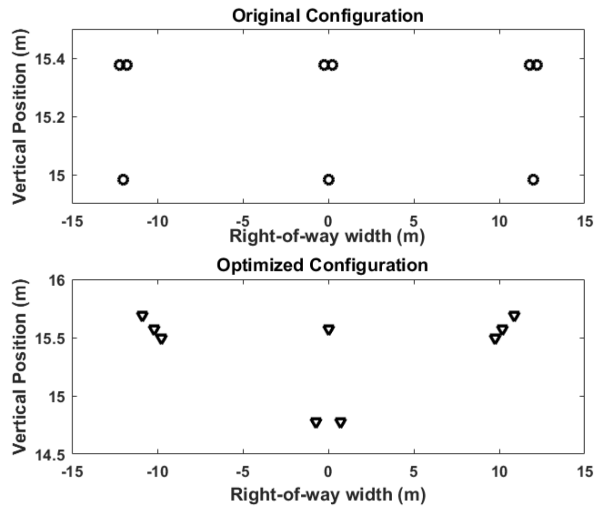

Figure 5. Original and Optimized Configuration - Case II - Three conductors

In this case, in Figure 5 the optimization process gives a solution with reduced distance between different phases and augmented the distance between conductors of the same phase. The initial and final positions of conductors in the case II can be analyzed from Table 5.

Table 5. Original and Optimized Positions

Case II	Phase 1 (m)	Phase 2 (m)	Phase 3 (m)
x_{orig}	-12.22 -12.00 -11.77	-0.22 0.00 0.22	11.77 12.00 12.22
x_{otim}	-10.85 -10.16 -9.75	-0.72 0.00 0.72	9.75 10.16 10.85
y_{orig}	15.37 14.98 15.37	15.37 14.98 15.37	15.37 14.98 15.37
y_{otim}	15.49 15.58 15.68	14.77 15.68 14.77	15.68 15.58 15.49

Similarly, in the case II is observed gain in the SIL near of 12.38% of the new bundle configuration, the growth is presented in Table 6.

Table 6. Case II - Surge Impedance Loading(SIL) and Surge Impedance(Z_c)

Case II	Original	Optimized
$Z_c(Ohms)$	276.28	246.60
$SIL(MW)$	904.85	1013.78
Increase SIL (%)	12.38	

7. ACKNOWLEDGMENT

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8. CONCLUSION

The Adjoint Method with a Gradient Method was applied to find optimal configurations of TL conductors to obtain lower values of the electric field at ground level. The increase in the SIL observed achieves near 6% and 12% in each case, respectively. The main trends of the optimized configurations are a decrease in the distances between different phases and an increase in the distances between the subconductors of a given phase. Such modifications are the basic actions to be taken in a High Surge Impedance Loading transmission line.

The obtained results show that the Adjoint Method can be used with other algorithms based on Gradient information, such as, Broyden–Fletcher–Goldfarb–Shanno (BFGS), Ellipsoidal Methods and others, working with elevate precision and faster than difference finite approximations.

The Sensitivity Analysis using the Adjoint Method has the potential to be used for more complex cases, with more than three conductors per phase.

REFERENCES

Acosta, J. and Tavares, M. (2018). Enhancement of overhead transmission line capacity through evolutionary computing. In *2018 IEEE PES Transmission & Distribution Conference and Exhibition-Latin America (T&D-LA)*, 1–5. IEEE.

Alexandrov, G.N. (1969). Theory of bundle conductors. *IEEE Transactions on Power Apparatus and Systems*, (6), 932–936.

Bakr, M. (2013). *Nonlinear optimization in electrical engineering with applications in matlab*. Institution of Engineering and Technology.

Bakr, M., Elsherbeni, A.Z., and Demir, V. (2017). *Adjoint sensitivity analysis of high frequency structures with Matlab*. The Institution of Engineering and Technology.

Balanis, C.A. (1998). *Advanced engineering electromagnetics*. John Wiley & Sons.

Dadash, M.S., Nikolova, N.K., and Bandler, J.W. (2012). Analytical adjoint sensitivity formula for the scattering parameters of metallic structures. *IEEE Transactions on Microwave Theory and Techniques*, 60(9), 2713–2722.

Director, S. and Rohrer, R. (1969). The generalized adjoint network and network sensitivities. *IEEE Transactions on Circuit Theory*, 16(3), 318–323.

EPRI, A. (2005). *Transmission Line Reference Book–200 kV and Above*. Electric Power Research Institute.

Glover, J.D., Sarma, M.S., and Overbye, T. (2012). *Power system analysis & design, SI version*. Cengage Learning.

Gomes Jr, S., Portela, C., and Fernandes, C. (1995). Principles and advantages of utilizing high natural power lines and presentation of comparative results - xiii snptee national seminar of production and transmission of electric energy. *Florianópolis, Brazil*.

Hall, J. and Deb, A.K. (1988). Prediction of overhead transmission line ampacity by stochastic and deterministic models. *IEEE transactions on power delivery*, 3(2), 789–800.

Kiessling, F., Nefzger, P., Nolasco, J.F., and Kaintzyk, U. (2014). *Overhead power lines: planning, design, construction*. Springer.

Lalau-Keraly, C.M., Bhargava, S., Miller, O.D., and Yablonovitch, E. (2013). Adjoint shape optimization applied to electromagnetic design. *Optics express*, 21(18), 21693–21701.

Luenberger, D.G., Ye, Y., et al. (1984). *Linear and nonlinear programming*, volume 2. Springer.

Maciel, R.d.P. et al. (2013). Maximização da potência característica de linhas de transmissão usando método de otimização não linear.

Melo, M.O., Fonseca, L.C., Fontana, E., and Naidu, S. (1999a). Electric and magnetic fields of compact transmission lines. *IEEE Transactions on Power Delivery*, 14(1), 200–204.

Melo, M.O., Fonseca, L.C., Fontana, E., and Naidu, S. (1999b). Electric and magnetic fields of compact transmission lines. *IEEE Transactions on Power Delivery*, 14(1), 200–204.

Nikolova, N.K., Bandler, J.W., and Bakr, M.H. (2004). Adjoint techniques for sensitivity analysis in high-frequency structure cad. *IEEE transactions on microwave theory and techniques*, 52(1), 403–419.

Paganotti, A.L., Afonso, M.M., Schroeder, M.A.O., Alipio, R.S., Gonçalves, E.N., and Saldanha, R.R. (2015). An adaptive deep-cut ellipsoidal algorithm applied to the optimization of transmission lines. *IEEE Transactions On Magnetics*, 51(3), 1–4.

Régis Jr, O., DART, F.C., CRUZ, A.L.P., and CHESF, C.C. (2009). Avaliação comparativa das concepções de linhas de potência natural elevada em 500 kv utilizadas no brasil. *XIII ERIAC*.

SALARI, J.C.; REIS, A. and ESTRELLA JR, L. (2019). Metodologia sistematizada para a otimização técnico-econômica da geometria dos feixes de condutores de linhas de transmissão aéreas. In *XVIII ERIAC*. CIGRE.

Sarma, M.P. and Janischewskyj, W. (1969). Electrostatic field of a system of parallel cylindrical conductors. *IEEE transactions on power apparatus and systems*, (7), 1069–1079.

Sarmiento, J.S.A. and Tavares, M.C. (2016). Enhancement the overhead transmission lines' capacity by modifying the bundle geometry using heuristics algorithms. In *2016 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, 646–650. IEEE.