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Reactive Power Control on a Consumer Unit with High Integration of PV Generation

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Abstract: The Federal University of Espírito Santo has acquired 5,441 kWp of solar photovoltaic distributed energy resource (DER), which is being installed on the rooftoop of the buildings of the Goiabeiras Campus in the city of Vitória, Espírito Santo, Brazil. Knowing that the solar plant would decrease the metered power factor, this paper analyzed whether the finance revenue of the DER installation may be increased by the injection of reactive power using the DER inverters. Results allow to state that the revenue may be lightly increased. Nevertheless, the costs of a real implementation are not addressed.

Keywords: Electric power systems, energy management, energy quality, photovoltaic systems, power factor.

1. INTRODUCTION

Due to the worldwide concern about reducing the emission of polluting gases, the interest of governments and society regarding the use of renewable sources for the electric energy generation is growing, especially when referring to distributed generation (Gan et al., 2007). Some factors, such as the relatively affordable price and the ease of installation, have made solar energy the most popular means of generating electricity using renewable sources (Blaabjerg et al., 2017; Adefarati and Bansal, 2016).

On February 11th, 2019, the Federal University of Espírito Santo (Ufes) contracted the installation of the largest distributed energy resource (DER) of the Brazilian state of Espírito Santo (Universidade Federal do Espírito Santo, 2018). The projected DER, composed by a 5,441 kWp Solar Plant, is bigger than the 5th largest Solar DER in operation in the whole country (Agência Nacional de Energia Elétrica, 2020).

Most of the solar plant is being installed on the rooftop of the buildings of the Goiabeiras Campus of Ufes. Located at the city of Vitória, Brazil, the campus is a medium-sized neighborhood composed by over a hundred of buildings fed by a 15 kV electrical energy distribution feeder under Ufes' crew administration. The internal grid is composed by over a hundred of medium voltage buses which feed seventy 15 kV/220 V Δ-Y transformers. The campus is supplied by the local electrical energy distributor, being billed by a meter installed on the connection point.

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Despite the many advantages of the photovoltaic generation (Roselli and Sasso, 2016; Apichonnabutr and Tiwary, 2018; Üçtuğ and Azapagic, 2018), the high integration of DER may give way to side effects which must be handed by the electrical system operator. The literature mentions some of them: changes in network levels of load current, in the grid power flow and in faults location and level (Walling et al., 2008; Balamurugan et al., 2012; Barker and Mello, 2000; Mendes, 2018; Mendes et al., 2018; Vargas et al., 2019).

The solar plant installed at Ufes injects a big amount of active power in the internal grid, which decreases the amount of demanded power from the external grid. Since all the reactive power demanded by the fed equipment keeps flowing from the external grid, the power factor seen by the distributor meter may be decreased. Knowing that Brazilian medium voltage fed facilities must keep a high power factor, the authors have previously applied the Brazilian regulation to evaluate the financial impacts of the extra reactive power charges caused by the DER (Pompermayer et al., 2019).

In Pompermayer et al. (2019), the authors studied the new power factor behavior of the Goiabeiras Campus, employing consumption data from February 22nd, 2015 to March 24th, 2017. Meteorological data from the same dates, acquired from a meteorological station located at the Campus, was used as well. The authors have estimated what would have been the generated active energy in each one-hour interval and compared to the consumed active and reactive energy.

A significant power factor decrease was observed during the day-light time, which suggests that the PV generation affects the metered power factor (Pompermayer et al., 2019). Despite this behavior, the authors concluded that the financial impacts are not as significant as it may seem. When the generated extra reactive power charges were compared to the raw economy provided by the DER employment, less than 91% of the one-hour intervals showed to cause extra reactive charges greater than 15% of the generated raw economy. None of the estimated proportional charges showed to be greater than 25% (Pompermayer et al., 2019).

Differently from Pompermayer et al. (2019), this paper aims to analyze whether it is suitable to control the facility power factor by generating reactive power using the DER inverters. The general objective of this paper is to analyze whether the financial revenue of Ufes Goiabeiras Campus due to the solar PV plants may be increased by the injection of reactive power using the DER inverters. The results are compared with the previously found results in order to provide an overview of the problem.

2. METHODOLOGY

The power factor limits in the state of Espírito Santo depends on the day hour. Between midnight and 6 a.m., any value of inductive power factor can be assumed, while a capacitive power factor must be greater than 0.92. After 6 a.m., any value of capacitive power factor can be assumed, while an inductive power factor must be greater than 0.92 (Agência Nacional de Energia Elétrica, 2010).

Whenever those limits are not observed, a charge is applied to the consumer unit. The applied extra reactive power charge is given by (1).

$$E_{RE_T} = EEAM_T \times \left(\frac{f_R}{f_T} - 1\right) \times VR_{ERE} \tag{1}$$

where E_{RE} is the applied charge for extra reactive power, $EEAM_T$ is the active energy metered during the T integration interval, f_R is the reference power factor (0.92), f_T is the metered power factor and VR_{ERE} is the extra reactive energy fee (Agência Nacional de Energia Elétrica, 2010).

For the extra reactive power charging purpose, an one-hour integration interval is applied in (1).

Equation (1) presents the extra reactive power charges as a function of the metered active energy. Since the metered active energy is the product of the metered active power (PM_T) and the integration time (t_T), and since the metered power factor is the ratio between the metered active power and the metered apparent power (SM_T), (1) may be written in the form of (2).

$$E_{RE_T} = V_{ERE} \cdot t_T \cdot (f_R \cdot SM_T - PM_T) \tag{2}$$

Equation (2) is written from the Distribution System Operator (DSO) point of view. This paper employs the consumer point of view which is more suitable to a consumer plant model. After this paradigm change, (2) may be written in the form of (3).

$$q_{cost} = c_q \cdot t_T \times \left(f_R \cdot s_{ext\_grid} + p_{ext\_grid}\right) \tag{3}$$

where q_{cost} is the extra reactive power cost, $c_q = V_{ERE}$ is the extra reactive power cost coefficient, $s_{ext\_grid} = SM_T$ is the apparent power flowing to the external distribution grid and $p_{ext\_grid} = -PM_T$ is the active power flowing to the external distribution grid.

Equation (3) is not equally applied all over the p_{ext_grid} spectrum. Despite the regulations in force in Brazil do not stating how the extra reactive power is billed when the facility is injecting power into the external grid ($p_{ext\_grid} > 0$), the local DSO do not apply any charge in this situation. Another discontinuity point in the extra reactive bill computation occurs when q_cost is negative. Whenever this occurs, the extra reactive charge is zero.

Since the apparent power of the DER inverters is up limited, the electrical system operator must hand a decrease in the generated active power when wishing the DER to generate reactive power. So, decreasing q_{cost} by means of DER reactive generation means increasing p_{cost}, the electrical energy cost. The electrical energy cost is proportional to the consumed electrical energy and is given by (4).

$$p_{cost} = -c_p \cdot t_T \cdot p_{ext\_grid} \tag{4}$$

where p_{cost} is the electrical energy cost, c_p is the electrical energy cost coefficient, t_T is the integration interval and p_{ext_grid} is the active power flowing to the external distribution grid.

The system operation cost calculation basis (z), ignoring collateral costs as maintenance and crew remuneration, is given by the sum of the extra reactive power cost and the electrical energy cost, which is expressed by (5).

$$z = p_{cost} + q_{cost} \tag{5}$$

The problem of knowing whether is suitable or not to generate reactive power in the DER inverters is an economic dispatch problem. An economic dispatch problem is an optimization problem in which the system operation cost must be minimized while keeping some operational constraints. This paper employs a simple economic dispatch methodology, by ignoring the energy losses in the internal system lines and concentrating all the generators and loads at a single bus. The system operation cost to be minimized is given by z in (5).

One of the constraints to be kept is the system power balance: the sum of all the powers flowing from the consumer bus is zero. The power balance is expressed by (6) and (7).

$$p_{ext\_grid} + p_{load} + p_{DER} = 0 \tag{6}$$
$$q_{ext\_grid} + q_{load} + q_{DER} = 0 \tag{7}$$

where p_{load} and q_{load} are, respectively, the active and reactive power flowing to the load; and p_{DER} and q_{DER} are, respectively, the active and reactive power flowing to the distributed energy resource.

The conceptual constraints are also to be kept, which means that the apparent powers must always be equal to the vectorial sum of the active and reactive powers. Equations (8) and (9) presents those constraints.

$$s^2_{ext\_grid} = p^2_{ext\_grid} + q^2_{ext\_grid} \quad (8)$$
$$s^2_{DER} = p^2_{DER} + q^2_{DER} \quad (9)$$

where s_{ext_grid} and s_{DER} are, respectively, the apparent powers flowing to the external grid and to the distributed energy resource; p_{ext_grid} and p_{DER} are, respectively, the active powers flowing to the external grid and to the distributed energy resource; and q_{ext_grid} and q_{DER} are, respectively, the reactive powers flowing to the external grid and to the distributed energy resource.

The DER inverters also impose some constraints: the apparent power is up limited (10) and the power factor is down limited (11).

$$s_{DER} \leq \overline{s_{DER}} \quad (10)$$
$$|p_{DER}| \geq \underline{pf_{DER}} \cdot s_{DER} \quad (11)$$

While a negative electrical energy cost is reverted as credit to the consumer unit, a negative extra reactive cost is not reverted, which makes unwanted to have such a situation. The constraint (12) avoids an over reactive power generation.

$$q_{cost} \geq 0 \quad (12)$$

The complete model is given by (3)-(12) and, from now on, is referred as the "min q_{cost} method".

The q_{cost} equation (3) is not equally applied all over the $p_{ext_{grid}}$ spectrum. In order to represent such a behavior, a discontinuous non-linear model should have been proposed, requiring the employment of a metaheuristc method for its solution. Metaheuristic methods are known for not assuring the very optimum solution of a problem (Blum and Roli, 2001). Nevertheless, as it will be further presented, the optimum behavior of the studied system in the discontinuous regions of q_{cost} is well known. This is why the proposed model does not represent those discontinuities, since a single comparison between the continuous model optimum results and those well known results is good enough to address the problem in the unrepresented regions.

In order to run the optimization routine, the mathematical modeling package JuMP has been employed. JuMP, as described by its presentation article: "is an open-source modeling language that allows users to express a wide range of optimization problems (linear, mixed-integer, quadratic, conic-quadratic, semidefinite, and nonlinear) in a high-level, algebraic syntax" (Dunning et al., 2017). JuMP is embedded in Julia, an open-source scientific computer language (Bezanson et al., 2017). The employed solver was the IPOPT, a primal-dual interior point algorithm for nonlinear programming (Wächter and Biegler, 2006).

3. RESULTS AND DISCUSSION

The developed model was implemented with the same estimated generation and consumption data used in Pompermayer et al. (2019). The data was collected between February 22nd, 2015 and March 24th, 2017, but the effect of a newly installed capacitor bank was added to the consumption registers.

The collected data is composed by 18,264 one-hour intervals. Employing the methodology of Pompermayer et al. (2019), from now on referred as "full generation method", 9,097 intervals would have registered any value of generated active power, totalizing 17.1 MWh of generated active energy. None of the intervals would have registered values of generated reactive power.

Figure 1 presents what the power factor of the Goiabeiras Campus of Ufes would have been at each hour of the day if the consumption pattern of the studied period happens again after the capacitor bank installation. The no generation power factor would have been a well-behaved data with an inter-quartile range varying from 0.017 to 0.030, which means that 50% of the power factor data is located within ±3% of the median value (Pompermayer et al., 2019).

Figure 2 presents what would have been the campus power factor with the same consumption pattern after the installation of the DER. A significant increase in the data variation may be seen within the interval from 7h to 17h, with the inter-quartile range varying from 0.02 to 0.30 (Pompermayer et al., 2019).

The raw economy (i.e. the economy provided by the generation, with no regards to extra reactive charges) provided by the full generation methodology would have reached 54.6% of the no generation energy bill. Nevertheless, the full generation methodology give way to an extra reactive bill which would have reached 0.67% of the no generation energy bill, decreasing the very economy to 53.9% of the energy bill.

Figure 3 presents what would have been the campus power factor employing the model in (3)-(12). The min q_{cost} power factor would have been a well-behaved data with an inter-quartile range varying from 0.000 to 0.079, which means that 50% of the power factor data is located within ± 8% of the median value. Compared to the behavior presented in Figure 2, the min q_{cost} power factor reveals that the min q_{cost} method effectively controls the DER reactive injection.

Nevertheless, employing the min q_{cost} method, the total active energy, generated during 9,064 one-hour intervals, would have been of 10.3 MWh, i.e. 39.3% lesser than the amount generated by the full generation method.

The raw economy provided by the min q_{cost} method would have reached 33.8% of the no generation energy bill, with a difference of 20.8% when compared to the raw economy provided by the full generation method. The extra reactive bill is not significant at all, what means that the very economy is also of the value of 33.8%.

The huge difference between the two methods would completely make the min q_{cost} method unfeasible. However, the ineffectiveness of the model in (3)-(12) may be explained by the discontinuities in the application of the q_{cost}, (3), in the extra reactive bill computation.

As aforementioned, (3) is not equally applied all over the p_{ext_grid} spectrum: when a facility is injecting power in the external grid ($p_{ext\_grid} > 0$), the local DSO do not apply any charge. Also, whenever q_{cost} is smaller than zero, the extra reactive charge is zero.

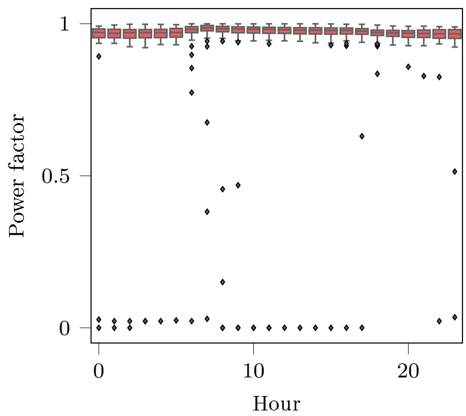

Figure 1. Power factor with no generation.
Source: Adapted from Pompermayer et al. (2019).

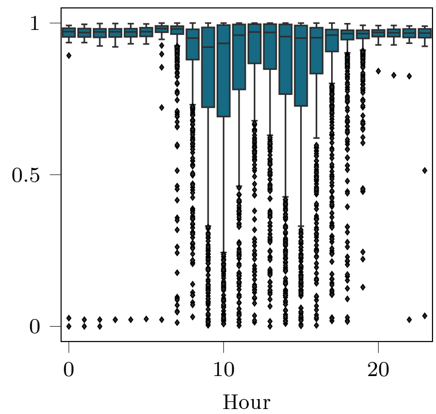

Figure 2. Power factor with full generation.
Source: Adapted from Pompermayer et al. (2019).

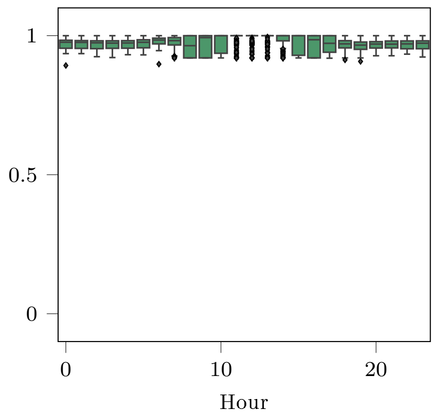

Figure 3. Power factor when minimizing the extra reactive cost.

In order to avoid a discontinuous non-linear optimization problem, the formulation in (3)-(12) do not model those two discontinuities, what give way to three different scenarios: when generating enough power to feed the load and to inject active power into the external grid; when the generation is not enough to feed the load, but the active power injection provokes extra reactive problems; and when the generation is not enough to feed the load and the active power injection do not provoke extra reactive problems. Figures 4a), 4b) and 4c) illustrates those three scenarios.

In the hypothetical scenario of Figure 4a), the load active power demand is of 400 kW, the load reactive power demand is of 300 kVAr and the DER apparent power is of 500 kVA. The green dashed line is the active energy cost (p_{cost}, see (4)), which decreases as the generated active power increases. The purple dashed line is the extra reactive cost (q_{cost}) as stated by (3). When the generated active power increases further than 400 kW, i.e. when there is injection of active power into the external grid, the q_{cost} value increases significantly. Nevertheless, the extra reactive bill, as computed by the local DSO (the solid purple line), do not follow the q_{cost} value, on the contrary, when the active power generation increases further than 400 kW, the extra reactive bill remains zero.

The orange dashed line presents z (see (5)), the sum of p_{cost} and q_{cost}. Since q_{cost} increases significantly when there is active power injection in the external grid, z increases too. On the other side, the orange solid line presents the sum of p_{cost} and the extra reactive bill as computed by the DSO. Since the extra reactive bill remains zero when power is injected to the external grid, the summed bill follow p_{cost}.

Since (3)-(12) doesn't comprise the discontinuities, the optimization algorithm precisely tracks the z optimum operation point (the green dot in Figure 4a), which is not the same as the effectively bill optimum operation point. Actually the full generation operation point (the red dot in Figure 4a) is the optimum operation point.

In the hypothetical scenario of Figure 4b), the load active power demand is of 500 kW, the load reactive power demand is of 120 kVAr and the DER apparent power is of 400 kVA. The green dashed line of the active energy cost also decreases as the generated active power increases. The purple dashed line of q_{cost} and the purple solid line of the extra reactive cost are very closed during all the generation spectrum.

The orange dashed and solid lines are also very closed to each other. As the optimization algorithm precisely tracks the z optimum operation point (the green dot in Fig. 4b), which is the same as the effectively bill optimum operation point, the algotithm behaves better than the full generation method (the red dot in Fig. 4b).

In the hypothetical scenario of Fig. 4c), the load active power demand is of 1000 kW, the load reactive power demand is of 200 kVAr and the DER apparent power is of 500 kVA. The green dashed line of the active energy cost also decreases as the generated active power increases. The purple dashed line of q_{cost} is always below zero. As aforementioned, when q_{cost} is smaller than zero, the applied extra reactive charge is zero. Thus, the purple solid line of the extra reactive cost are zero during all the generation spectrum.

The orange dashed line is slightly below the solid line during all the generation spectrum. The optimization algorithm precisely tracks the z optimum operation point (the green dot in Figure 4c), which is not the same as the effectively bill optimum operation point. Actually the full generation operation point (the red dot in Figure 4c) is the optimum operation point.

The two discontinuities found in the extra reactive bill computation make the extra reactive charges to assume zero value, what leads the total bill to be just equal to the energy bill. Since the energy bill is inversely proportional to the generated active power, in those discontinuous regions, the optimum operation point is the full generation. In the region where (3) applies to the extra reactive bill computation, the min q_{cost} method is able to find the optimal solution.

Thus, the min q_{cost} method may be improved by the addition of a supervision layer. Such a layer compares the economy provided by the full generation method at each

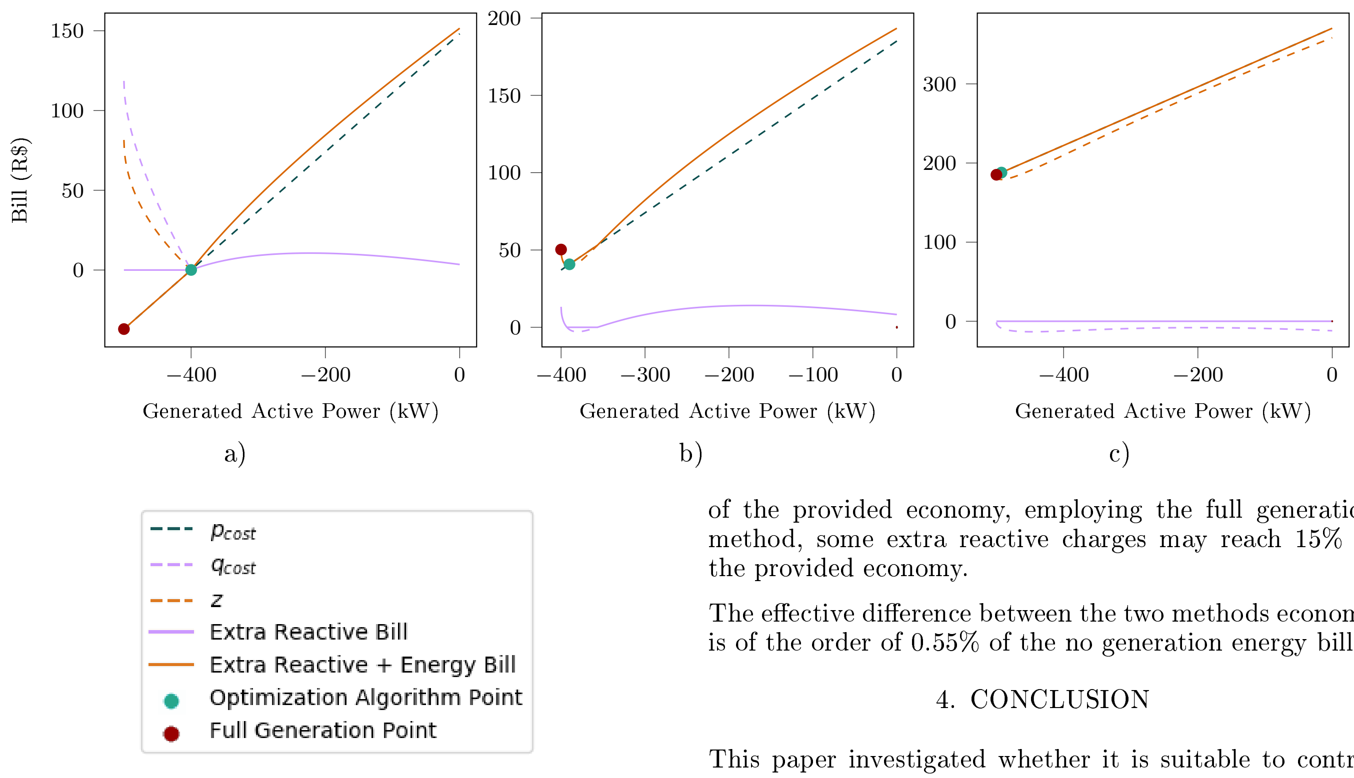

Figure 4. Enough generation (a) Not enough generation (b) Not enough generation with no extra reactive problems (c)

one-hour interval with the economy provided by the min q_{cost} method, choosing always the bigger one.

Figure 5 presents the power factor when applying the improved method. In this figure it may be seen that the power factor still being a well-behaved data like the one obtained from the min q_{cost} method, but presents some features of the full generation method, like the presence of small outliers.

Employing the improved method, the total active energy, generated during 9,080 one-hour intervals, would have been of 17.0 MWh, i.e. 0.22% lesser than the amount generated by the full generation method.

The raw economy provided by the min q_{cost} method would have reached 54.5% of the no generation energy bill, with a not significant difference when compared to the raw economy provided by the full generation method. The extra reactive bill is not significant at all, what means that the very economy is also of the value of 54.5%.

The difference between the results from the application of the full generation method and the improved method is illustrated in Figure 6.

The y-axis of the Figure 6 presents the extra reactive bill in percentage of the raw economy provided by each one-hour interval in the studied data set. The x-axis presents the percentage occurrences of one-hour intervals presenting an extra reactive value equal or small to the value in the y-axis.

While in the improved method, 90% of the one-hour intervals presents an extra reactive charge smaller than 2.2% of the provided economy, employing the full generation method, some extra reactive charges may reach 15% of the provided economy.

The effective difference between the two methods economy is of the order of 0.55% of the no generation energy bill.

4. CONCLUSION

This paper investigated whether it is suitable to control the facility power factor by generating reactive power in the DER inverters. The study employed data from Pompermayer et al. (2019) to evaluate what would be the campus power factor with the DER power factor control.

Results allow to state that the power factor may be effectively regulated and the financial revenue may be increased. Nevertheless, in the studied case the effective difference between the two methods economy would have been of the order of 0.55% of the no generation energy bill.

To state whether it is suitable or not to control the power factor by using the DER still a matter of study. It is important to know what would be the operational cost of such technique and whether the small difference presented in the results would be worth of the implantation and operation effort.

Future works may consider to employ a bigger range of data or even stochastic methods to better test the solution as well as to be able to predict the future behavior. The radiation pattern, as well as the consumption pattern, may be represented as a random variable and the power factor, as well as the proportional charges, may be modeled by a density probability function.

Goiabeiras Campus of Ufes has its own distribution electrical feeder. This paper concentrated all the loads and generator into a single bus. It's also important to observe how the DER affects the power factor all over the feeder.

It may be also important to observe the effects of the DER reactive control over a more reactive data, such as the Goiabeiras Campus of Ufes with no capacitor bank compensation.

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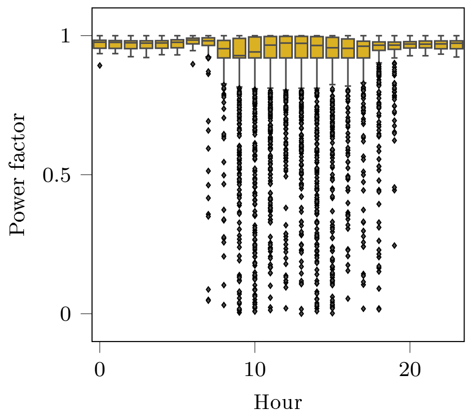

Figure 5. Optimum generation power factor.

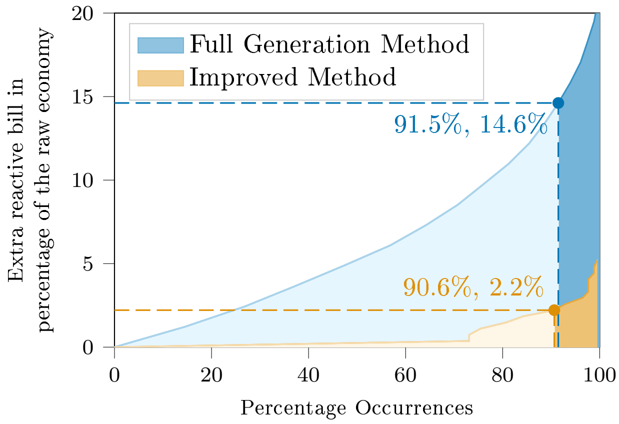

Figure 6. Cumulative occurrences of extra reactive bill

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