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A Stochastic Programming Model for the Optimal Allocation of Photovoltaic Distributed Generation in Electrical Distribution Systems Considering Load Variations and Generation Uncertainty

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Abstract: Nowadays, the penetration of distributed generation (DG) units in power systems is increasing because of their benefits on the power systems. Place, type and size of distributed generators play an important role in power loss reduction, power quality improvement, security enhancement, and cost reduction. Therefore, optimal placement and sizing of DG units in electric power systems are one of the most important problems that should be evaluated carefully. DG allocation is a constrained optimization problem with different important objectives such as power loss minimization, voltage profile improvement, reliability enhancement, investment and operation cost reduction, etc. In this paper, regarding higher distribution active losses compared to transmission and generation losses and investment limitation, DG allocation problem is solved for photovoltaic units, aiming minimization of energy and investment costs considering generation uncertainty and load variation. Due to high uncertainties of solar energy resource, the problem is evaluated under different scenarios of solar radiation under a stochastic programming approach. Tests were carried out using the 33-node distribution system and the obtained results demonstrate the advantage of optimal DG allocation as well as the efficiency of the adopted mathematical to find the optimal solution.

Keywords: Distributed Generation; Stochastic modeling; Generation uncertainty; Load variation; Optimal allocation.

NOMENCLATURE

Sets:

Ω_b: Set of buses.
Ω_d: Set of load levels.
Ω_g: Set of DG.
Ω_l: Set of branches.
Ω_t: Set of scenarios.

Parameters:

$\alpha_{d,t}$: Number of days in one year of scenario t in load level d (h).
CDG_g Energy cost for DG of type g (\$/kWh).
c_g Annualized installation cost of DG unit g (\$).
CS Energy cost of energy supplied by the substation (\$/kWh).
f_t^{DG} Generation factor of DG in scenario t.
I_{ij}^{max} Maximum current magnitude of branch ij (A).
$N_{DG}^{\max}$ Maximum number of DG units.
$N_{DG,g}^{\max}$ Maximum number of DG of type g.
$P^D_{i,d}$ Active power demanded at node i in load level d (kW).
P_g^{max} Maximum active power provided by DG of type g (kW).
Pr_t Probability of scenario t.
pf^{DG} Power factor limit for DG.
pf^S_{min} Minimum leading and lagging substation power factor.
Q_g^{max} Maximum reactive power provided by DG of type g (kVAr).
Q_g^{min} Minimum reactive power provided by DG of type g (kVAr).
$Q^D_{i,d}$ Reactive power demanded at node i in load level d (kVAr).
R_{ij} Resistance of branch ij (Ω).
V_{min} Minimum voltage magnitude (kV).
V_{max} Maximum voltage magnitude (kV).
X_{ij} Reactance of branch ij (Ω).
Z_{ij} Impedance of branch ij (Ω).

Variables:

$I_{ij,d,g,t}$ Current magnitude of branch ij at load level d in scenario t (A).
$I_{ij,d,g,t}$ Current magnitude of branch ij at load level d in scenario t (A).
$P^{DG}_{i,d,g,t}$ Active power provided by DG of type g on bus i at load level d in scenario t (kW).
$P^{S}_{i,d,g,t}$ Active power provided by substation on bus i at load level d in scenario t (kW).

$P_{ij,d,g,t}$ Active power of branch ij at load level d in scenario t (kW).
$Q^{DG}_{i,d,g,t}$ Reactive power provided by DG of type g on bus i at load level d in scenario t (kVAr).
$Q_{ij,d,g,t}$ Reactive power of branch ij at load level d in scenario t (kVAr).
$Q^{S}_{i,d,g,t}$ Reactive power provided by substation on bus i at load level d in scenario t (kVAr).
$V_{i,d,g,t}$ Voltage magnitude on bus i at load level d in scenario t (kV).
$y_{i,,g}$ Integer decision variable for DG unit g (it is equal to 1 if DG unit g is installed, otherwise it will be 0).

Functions:
C_T Total cost ($).
C_{DG} Cost of generated energy by DG units ($).
C_S Cost of generated energy by substation ($).
IC Investment cost ($).

1. INTRODUCTION

The distribution networks are responsible for supplying the energy demand of the end users of the electrical system (Badran et al. 2017; Paterakis et al. 2016), under standards of service quality at the time that guarantees an economic operation (Duttagupta and Singh 2006). Given the large number of users in this portion of the electrical system, each of the previous tasks becomes a huge engineering challenge.

To meet the energy requirements of the consumers, the distribution companies try to have a wide margin for all possible fluctuations in the demand profile. This flexibility cannot come only from the transmission system, where an increase in energy production presupposes large investments, but can be obtained by using distributed generation (DG), specifically that of a renewable nature which as the great advantage of aiding to reduce emissions of greenhouse gases (Braslavsky et al. 2014).

On the economic operation side, distribution companies are mainly concerned with reducing losses. In the distribution system most of the losses occur determining, hence, the total efficiency of the power system. These losses result in additional costs for the companies, which cannot be totally eliminated but minimized (Rahmani-Andebili and Fotuhi-Firuzabad, 2018).

In order to minimize the losses, several techniques are used, such as reconduction, installation of capacitor banks, voltage regulators, reconfiguration of the distribution network and, in modern times, distributed generation (Acharya et al. 2006). Among these techniques, the option for distributed generation presents the greatest simultaneous benefits for the distribution network. Among these additional benefits are the increased flexibility to meet demand and reduction of power losses, as previously mentioned. In addition, DG improves the voltage profile and reliability of the system (Letsela et al. 2002), whenever the installation of distributed generation follows a planning process sizing and allocating it in the network (Aman et al. 2013).

Therefore, the DG allocation (DGA) is very important for optimization of the distribution network operation (Hamidi and Chabanloo, 2018). DGA is an optimization problem with many technical and operational constraints that have been solved using classic and metaheuristic methods (Ganguly and Samajpati, 2015; Ali et al. 2017; Kansal et al. 2013). Since the DGA problem was proposed, classic optimization methods have been among the most important techniques to solve it; they are widely used due to their convergence and optimization characteristics. The metaheuristics have been widely used to solve the DGA problem in large-scale systems in which classic optimization methods are time consuming. However, these approximate techniques do not provide information regarding the optimal or the optimization process (Ali et al. 2017). Also, the parameters in each metaheuristic are conveniently determined for each case to be solved, which does not allow their application in a standard way, even in the same optimization problem.

Acharya et al. (2006) proposed an analytical technique based on sensitivity analysis of power losses to solve DGA problem aiming power losses minimization. In this approach, first, the optimal size of DG on each bus was determined using an exact nonlinear power losses formulation. Then, the optimal DG allocation was identified using a linear approximated value of power losses. This methodology results useful for small instances of DGA problem, but on the large-scale it leads to computational inefficiency related to the impedance matrix calculation.

In order to improve the proposal in (Acharya et al. 2006), (Hung et al. 2010), it is included reactive power of DGs injections into Acharya's formulation. Simulation results proved to be better than his predecessor. However, this approach cannot be employed to DG allocation in large-scale distribution networks, because of computational limitations of analytical methods in solving nonconvex optimization problems. In the same year, Ghosh et al. (2010) proposed a simple Newton-Raphson based search method to find the best DG allocation considering the minimization of power losses and DG operation cost. Recently, in (Viral and Khatod, 2015), network losses were minimized by DG allocation using an analytical technique, too. The numerical results show that the total number of load flow does not increase with the size of the system, but the proposed method cannot be applied to meshed distribution networks.

In addition, (Murty and Kumar, 2015) defined a new index known as voltage stability for the DGA problem that aims active and reactive losses minimization taking into account different load factors. Simulation results indicated that DG with lagged power factor decreases power losses. Furthermore, Mahmoud et al. (2016) presented an efficient method for losses minimization through the DGA considering different types of DG. The approach made an integration of analytical method with optimal power flow, where the results are better than other analytics techniques.

In (Mena and García, 2015), an efficient MINLP approach was presented to solve DGA problem considering network losses and generation cost of both transmission and distributed generators. In this method, the problem is divided into two sub-problems: allocation and size of generation. At first, the allocation subproblem is resolved while the optimal size of each allocated DG is determined in the second subproblem.

In (Ganguly and Samajpati, 2015), an adaptive genetic algorithm (GA) was proposed to solve a stochastic DGA problem aiming power losses minimization of radial distribution systems under load and generation uncertainties. The objective function considers a weighted sum of the minimization of power losses and voltage deviation. Results show that the fuzzy-based method is efficient to deal with load growth. Another metaheuristic was proposed in (Ali et al. 2017), where authors solved a stochastic DGA model which minimizes the network power losses and maximizes voltage stability through the allocation of renewables DGs using ant lion optimization algorithm. Numerical results show that this algorithm can reduce the losses and enhance voltage profile effectively.

Rueda-Medina et al. (2013) and Melgar Dominguez et al. (2018) formulated a mixed integer linear programming (MILP) for DGA problem using AMPL (Fourer et al. 2003) and solved it with CPLEX. In (Rueda-Medina et al. 2013), the investment and operational costs of DG as well as active power losses were minimized by optimal placement and sizing of DGs considering load variations and short-circuit level. Most recently, Melgar Dominguez et al. (2018), minimized the cost of energy supplied by substations and the investment in DG as well as energy storage systems (ESSs) and capacitor banks allocation cost.

Although DGA problem has been solved using different approaches, such as analytical, classical optimization, and metaheuristics, further research on mathematical modeling must be done in order to explore how to properly consider uncertainties. Therefore, in this paper is proposed a mixed-integer second order conic programming formulation for the DGA problem. The formulation considers investment and generation cost of DG units as well as the cost of the energy supplied by substations, under load variations and generation uncertainties. That stochastic mathematical model is written in AMPL and solved using CPLEX.

The paper is organized as follows. In Section 2, the problem formulation is described. In section 3, the proposed model is evaluated using one test system considering photovoltaic units. Finally, the conclusion is presented in section 4.

2. PROBLEM FORMULATION

The proposed DGA model is formulated as an instance of stochastic programming, where the correlated uncertainty of photovoltaic generation is characterized by a set of scenarios. The proposed model is built on the deterministic formulation wherein: (i) the daily load curve is discretized into several load levels, (ii) an exactly network model is used, (iii) the costs of generation from substation and DG are included in the objective function, and (iv) several investment alternatives for DG are considered.

The AC power flow model represented in Fig 1. illustrates the effect of the distribution network into DGA problem using active and reactive power flows through branches ($P_{ij,d,g,t}$ and $Q_{ij,d,g,t}$), branch currents ($I_{ij,d,g,t}$), active and reactive power generated by DG units ($P^{DG}_{ij,d,g,t}$ and $Q^{DG}_{ij,d,g,t}$) and substation ($P^{S}_{i,d,g,t}$ and $Q^{S}_{i,d,g,t}$), and nodal voltages ($V_{i,d,g,t}$). The decision variables $y_{i,g}$ are integer variables for the number, type g, and allocation in bus i.

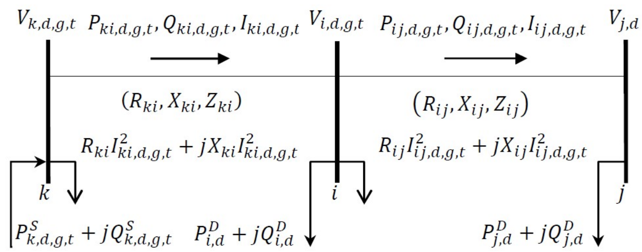

Fig. 1: Example network.

$$\min C_T = IC + C_{DG} + C_S \tag{1}$$

where,

$$IC = \sum_{i\in\Omega_b}\sum_{g\in\Omega_g} c_g y_{i,g} P_g^{max} \tag{2}$$

$$C_{DG} = \sum_{t\in\Omega_t}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} \alpha_{d,t} P^{DG}_{i,d,g,t} CDG_g \Pr_t \tag{3}$$

$$C_S = \sum_{t\in\Omega_t}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} \alpha_{d,t} P^{S}_{i,d,g,t} CS \Pr_t \tag{4}$$

Subject to:

$$\sum_{ki\in\Omega_l} P_{ki,d,g,t} - \sum_{ij\in\Omega_l}\left(P_{ij,d,g,t} + R_{ij} I^2_{ij,d,g,t}\right) + P^S_{i,d,g,t} + \sum_{g\in\Omega_g} P^{DG}_{i,d,g,t} = P^D_{i,d} \tag{5}$$

$$\sum_{ki\in\Omega_l} Q_{ki,d,g,t} - \sum_{ij\in\Omega_l}\left(Q_{ij,d,g,t} + X_{ij} I^2_{ij,d,g,t}\right) + Q^S_{i,d,g,t} + \sum_{g\in\Omega_g} Q^{DG}_{i,d,g,t} = Q^D_{i,d} \tag{6}$$

$$V^2_{i,d,g,t} - 2\left(R_{ij}P_{ij,d,g,t} + X_{ij}Q_{ij,d,g,t}\right) - \left(R^2_{ij} + X^2_{ij}\right) I^2_{ij,d,g,t} - V^2_{j,d,g,t} = 0 \tag{7}$$

$$V^2_{j,d,g,t} I^2_{ij,d,g,t} = P^2_{ij,d,g,t} + Q^2_{ij,d,g,t} \tag{8}$$

$$V^2_{\min} \le V^2_{i,d,g,t} \le V^2_{\max} \tag{9}$$

$$0 \le I^2_{ij,d,g,t} \le \left(I^{\max}_{ij}\right)^2 \tag{10}$$

$$0 \le P^{DG}_{i,d,g,t} \le f^{DG}_t P^{\max}_g y_{i,g} \tag{11}$$

$$Q^{\min}_g y_{i,g} \le Q^{DG}_{i,d,g,t} \le Q^{\max}_g y_{i,g} \tag{12}$$

$$-P^{DG}_{i,d,g,t}\tan(\cos^{-1}(pf^{DG})) \le Q^{DG}_{i,d,g,t} \le P^{S}_{i,d,g,t}\tan(\cos^{-1}(pf^{DG})) \tag{13}$$

$$-P^{S}_{i,d,g,t}\tan(\cos^{-1}(pf^{S}_{\min})) \le Q^{S}_{i,d,g,t} \le P^{S}_{i,d,g,t}\tan(\cos^{-1}(pf^{S}_{\min})) \tag{14}$$

$$0 \le \sum_{i\in\Omega_b}\sum_{g\in\Omega_g} y_{i,g} \le N^{\max}_{DG} \tag{15}$$

$$0 \le y_{i,g} \le N^{\max}_{DG,g} \tag{16}$$

$$\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} y_{i,g}P_g^{\max} \le \beta \max\left\{\sum_{i\in\Omega_b} P_{i,d}^D\right\} \tag{17}$$

The objective function minimizes the total annual operation cost of distribution system that includes the DG investment cost (1), energy cost supplied by DG (2) and energy cost supplied by substations (3).

The constraints associated with the system operation are formulated by (5)−(8). Equations (5) and (6) describe the first Kirchhoff's law by the active and reactive nodal power balance in presence of DG, respectively. Equation (7) represents the second Kirchhoff's law which is related to voltage drop. Finally, (8) explains the relationship between apparent power of each branch (left-hand-side of expression) and its active and reactive power flow (right-hand-side of expression).

The operating limits are expressed by (9)−(14), where (9) limits the variation of the square of the voltage and (10) limits the current flow thought each branch in the network. In addition, (11) and (12) limit the active and reactive power generation of each DG based on its size. Furthermore, (13) limits the reactive power of DGs based on the permissible power factor and (14) limits the reactive power of substation based on the permissible power factor.

Investment decisions are constrained according to the following expressions. Expressions (15) and (16) show maximum numbers of DG that can be installed in network and maximum numbers of each DG type that can be installed on each bus because of investment and technical limitations, respectively. Expression (17) limits the penetration of DG active power, i.e., the maximum active power provided by DG which must be less or equal to a fraction ($0<\beta\le1$) of total peak load of the system.

The above MINLP formulation cannot be solved by convex commercial tools because of non-linear terms as $I^2_{ij,d,g,t}$ and $V^2_{i,d,g,t}$. The variable change method is used to reformulate the problem by replacing square variables $I^2_{ij,d,g,t}$, $V^2_{j,d,g,t}$ and $V^2_{i,d,g,t}$ with $I^{sqr}_{ij,d,g,t}$, $V^{sqr}_{j,d,g,t}$, and $V^{sqr}_{i,d,g,t}$, respectively. Additionally, expression (8) is non-linear but can be recast in a second order constraint as showing in (22).

Therefore, the above MILNP can be recast as a second-order conic programming problem as show in (11)−(17) and (18)−(21). This convex formulation ensures that optimal solutions can be obtained and can be solved by commercial solvers such as CPLEX.

$$\begin{aligned} C_T = &\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} c_g y_{i,g} P_g^{\max} + \sum_{t\in\Omega_t}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} \alpha_{d,t} P^S_{i,d,g,t} CS\,\mathrm{Pr}_t \\ &+ \sum_{t\in\Omega_t}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} \alpha_{d,t} P^{DG}_{i,d,g,t} CDG_g\,\mathrm{Pr}_t \end{aligned} \tag{18}$$

Subjected to (11)–(17) and:

$$\begin{aligned} &\sum_{ki\in\Omega_l} P_{ki,d,g,t} - \sum_{ij\in\Omega_l}\left(P_{ij,d,g,t} + R_{ij} I^{sqr}_{ij,d,g,t}\right) + P^S_{i,d,g,t} \\ &+ \sum_{g\in\Omega_g} P^{DG}_{i,d,g,t} = P^D_{i,d} \end{aligned} \tag{19}$$

$$\begin{aligned} &\sum_{ki\in\Omega_l} Q_{ki,d,g,t} - \sum_{ij\in\Omega_l}\left(Q_{ij,d,g,t} + X_{ij} I^{sqr}_{ij,d,g,t}\right) + Q^S_{i,d,g,t} \\ &+ \sum_{g\in\Omega_g} Q^{DG}_{i,d,g,t} = Q^D_{i,d} \end{aligned} \tag{20}$$

$$\begin{aligned} &V^{sqr}_{i,d,g,t} - 2\left(R_{ij}P_{ij,d,g,t} + X_{ij}Q_{ij,d,g,t}\right) - \left(R_{ij}^2 + X_{ij}^2\right) I^{sqr}_{ij,d,g,t} \\ &-V^{sqr}_{j,d,g,t} = 0 \end{aligned} \tag{21}$$

$$V^{sqr}_{j,d,g,t} I^{sqr}_{ij,d,g,t} \ge P^2_{ij,d,g,t} + Q^2_{ij,d,g,t} \tag{22}$$

$$V^2_{\min} \le V^{sqr}_{i,d,g,t} \le V^2_{\max} \tag{23}$$

$$0 \le I^{sqr}_{ij,d,g,t} \le \left(I^{\max}_{ij}\right)^2 \tag{24}$$

3. CASE STUDY

This section presents and discusses results from one case of study in order to verify the efficiency of the proposed model. The proposed DGA problem was applied to 33-bus distribution system using CPLEX in AMPL. The simulations have been implemented on a Dell computer with 64-bit processor and 3.6GHz Intel core i7. All data related to network system is given in (Baran and Wu, 1989) where the limits of $V_{\min}$ and $V_{\max}$ are 0.9 and 1.0 p.u. for load nodes, and voltage magnitude of substation nodes are fixed in 1 p.u. Finally, energy cost for substation is 0.2 \$/kWh (Rueda-Medina et al. 2013).

According to Fig. 2, all DGs are considered to be photovoltaic units with four generation profiles in order to represent uncertainty in generation, where each scenario indicates situation of solar irradiation. For example, scenario 4 shows the minimum generation of photovoltaics DG which may represent a cloudy day, while scenario 1 shows the maximum generation of photovoltaic DG. Probability of each scenario is 0.25. Besides, different load levels are considered in order to taking into a count the hourly load variation as showing in Fig. 3.

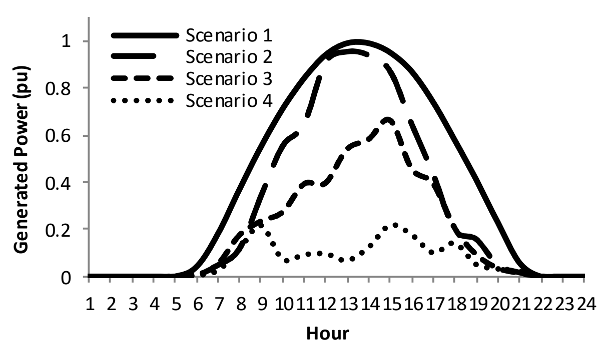

Fig. 2 Photovoltaic generation profile for each scenario (Franco et al. 2018).

The solution time required by AMPL/CPLEX to solve the model for the 33-bus test system was 4.43 min. Table 1 lists capacity, energy cost and investment of different DG types for both case studies. Also, energy costs of DG which is related to operation and maintenance costs for types 1 and 2 are 21 \$/kW-year and 19 \$/kW-year respectively.

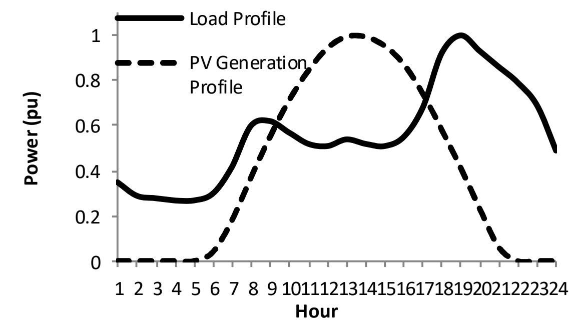

Fig. 3 Normalized load and PV generation profiles (Franco et al. 2018).

Table 1. Capacity, and installation and energy costs for different DG types (Theo et al. 2017)

Type	Max. Active Power (kW)	Reactive power (kVar)		Installation cost ($/kW-yr)	Energy cost ($/kWh)	Max. No. of DG on each bus
		Max.	Min.			
1	10	4	-4	194.85	0.0024	10
2	100	40	-40	173.15	0.0022	5

The problem was simulated regardless of DG units, in which the worst voltage magnitude was 0.9126 at bus 18. Also, the proposed model was applied in presence of photovoltaic generation and the minimum voltage magnitude was 0.913 at bus 17. Table 2 shows the results about selected buses for installation of each type of DGs, number of installed DG units and their generation. Along with, Table 3 describes investment and cost of energy from DGs and substation. Finally, generation at substation bus and network losses are compared in Figs 4 and 5 before and after DG allocation, respectively.

Table 2. Capacity, and installation and energy costs for different DG types

Bus	Number		Annual Generation in Every Scenario on Each Bus (kWh)			
	Type 1	Type 2	t=1	t=2	t=3	t=4
10	0	3	1835.1855	1334.9665	874.7533	296.4682
14	0	2	1220.7662	889.2332	583.5479	197.7830
16	0	1	611.4360	445.6679	292.3777	99.0231
17	1	0	61.1149	44.4947	29.2308	9.9481
18	10	1	1233.0774	899.8217	595.1856	198.5696
30	0	5	3048.6457	2219.9617	1457.6362	494.0236
31	0	2	1219.6136	888.1364	583.2068	197.6924
32	0	2	1219.6139	888.1388	583.2144	197.6980
33	4	1	840.2529	621.8634	408.4797	138.5386

From Table 2, one, ten and four DG units with capacity of 10 kW are installed on buses 17, 18 and 33, respectively. Also, one 100-kW DG unit on buses 16 and 33, two 100-kW DG units on buses 14, 31 and 32, and three and five 100-kW DG units on buses 10 and 30 should be installed, respectively.

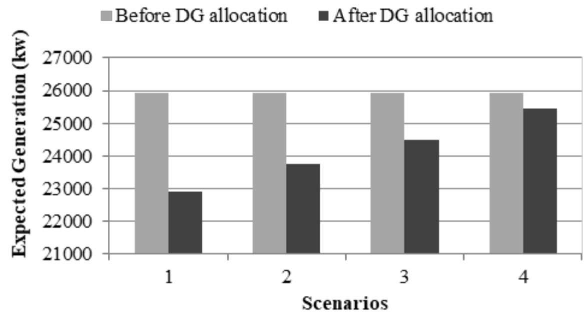

Fig. 4 Generation at substation bus

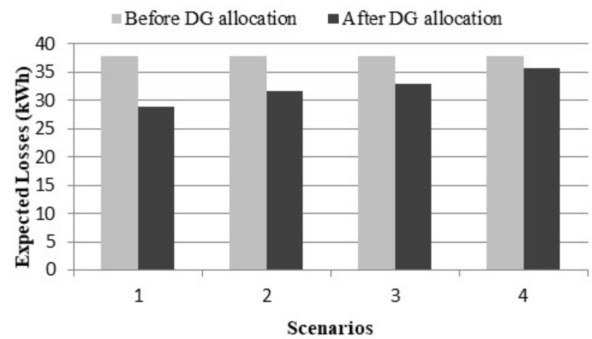

Fig. 5 Active power losses.

Table 3. The investment and operation costs network (10^3 $)

Costs	Before DG allocation	After DG allocation
IC^1	0	323.5825
$C_{DG}{}^2$	0	5.41151
$C_S{}^3$	7575.52037	7049.45874
$C_T{}^4$	7575.52037	7378.45275

[1]Investment cost, [2]Cost of energy generated by DGs, [3]Cost of energy generated by substation, [4]Total cost

According to Fig. 4, total power generated at substation bus is reduced in 6.9% after DG allocation. It means that rest of generation is compensated by DG units. As other result of the DGA problem, the total power losses are also reduced in 14.65% when compared with the case without DG, as illustrate in Fig. 5.

The results in Table 3 indicate that the installation of DG units requires the investment and operation costs of DG units, $323582.5 and $5411.51, respectively. Fortunately, the DGs installation decreases the cost of energy supplied by substation in $526060 (6.94%) and, therefore, saving $197070 in total cost.

4. CONCLUSIONS

Optimal allocation of distributed generation is very important for ensuring the operational conditions of the distribution network. Power generation of photovoltaic DG units is variable and depends on sun radiation, which is a probabilistic event. Therefore, a stochastic formulation is presented for the optimal allocation of DG units in distribution networks under load variations and generation uncertainties. The daily load variations and different scenarios for solar generation are considered in the proposed model. The objective is to minimize cost of energy generated by substation as well as investment and operation costs of distributed generators.

The proposed DGA problem is a constrained mixed-integer conic optimization problem. The decision variables are number, type and allocation of DG units as well as real variables active and reactive powers of branches, branch currents, active and reactive power generated by DG units and substation and nodal voltages.The results show that although employing DG units requires the investment and operation cost of distributed generators, the cost of energy generated by substation is reduced, decreasing total cost of network. In other words, although DG installation imposes investment and operation costs of distributed generators to network, it leads to cost savings. DG units can reduce network losses by generating the power at the load points and therefore they reduce the network costs.

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