Direct Extremum-Seeking Control applied to a quadruple tank system

Rayra Destro * **Gabriel Pereira das Neves** * **Claudio Garcia** *
Bruno Augusto Angélico *

* *Department of Telecommunications and Control Engineering, Escola Politécnica da USP, São Paulo, Brasil, (e-mail: rayra.destro@usp.br, gabriel.pereira.neves@usp.br, clgarcia@lac.usp.br, angelico@lac.usp.br)*

Abstract: This article presents a model-free control technique based on an extremum seeking control algorithm (ESC). The idea is to use ESC as a direct controller to minimize the quadratic error. Considering that it is possible to minimize the error to zero, it is then possible to track a reference. In addition, the technique can guarantee operation for different points of application, as well as for time-varying parameters. This technique was tested, through simulations, in a quadruple tank system.

Keywords: Extremum Seeking Control; Model-free control; Quadruple tank; Online optimization.

1. INTRODUCTION

Different industrial and academic systems present high complexity in obtaining their models. To meet this need, different kinds of control techniques were developed over time. Techniques such as adaptive control (Dumont and Huzmezan, 2002), algebraic estimators (Fliess and Sira-Ramírez, 2003; Fliess and Join, 2008), reinforcement learning (Zhang et al., 2019), ESC (Ariyur et al., 2003) and others have been developed for this propose. This work proposes a direct application in tracking reference using the ESC technique.

To present an application of a model-free control technique based on an ESC algorithm the plant chosen for this work is the quadruple tank system. It is a MIMO (multiple-inputs multiple-outputs) system such as most part of the industrial plants. The system is composed of four interconnected tanks, 2 valves and 2 pumps. The sum of the percentages of the 2 valves opening defines a particularity of this system: an adjustable zero. If the sum is greater than 1, the system has a minimum phase zero and, if the sum is lower than 1, it has a non minimum phase zero. Another characteristic of this system is that it has a considerable coupling between channels. Because of these features, the quadruple tank is widely studied and different kind of control techniques have been implemented for it.

A static decoupling is used in (Astrom et al., 2002) to explain how the system works with a decoupled PI. (Mehri and Tabatabaei, 2021) uses sliding mode control and (Li and Zheng, 2014) employs H_∞ to control the system. Besides, techniques such as LQG/LTR (das Neves et al., 2016) and QFT control applied with a dynamic decoupler (das Neves and Angélico, 2016) were tested on this plant.

Herewith, this article proposes the use of an optimizer that works in real time that can increase the plant performance without a mathematical model of the system. ESC is a model-free control technique, that has been in evidence lately. For a complex plant, for example, it is not practical to try to design a mathematical model and a practical way of bypassing this need is to use a model-free control when trying to improve the system's performance.

Though the quadruple tank is not a complex system, it has the particularities shown above and its behavior is well known, what makes it a good system for testing and validating the quality of ESC as a model-free control.

The article proposes to use the ESC technique not for tunning a controller as in (Killingsworth and Krstic, 2006) but to directly calculate the control effort considering different references.

It is valid to mention that the proposed model-free control is not associated to an operational point. In other words, the control is valid in a larger range than a classical linear control technique.

This article is divided as follows. Firstly, Section 2 presents the stability demonstration for ESC. In Section 3 is described the non-linear model of the quadruple tank system. We stress that this model is used only for simulation, the model was not used for the controller design. Section 4 presents the controller scheme design. Section 5 presents the results obtained for different references and valve openings. Finally, the conclusions of this paper are presented in Section 6.

2. EXTREMUM SEEKING CONTROL

The extremum seeking is an analog optimizer that works in real time (Krsti and Wang, 2000). This algorithm uses only the output measurement to calculate the input and it makes the output to converge to an optimum point.

There are some techniques to implement ESC, such as gradient, sliding-mode and the most common is by sinusoidal disturbance, which is considered in this work (Krsti and Wang, 2000; Zhang and Ordóñez, 2012).

To prove the algorithm, consider the block diagram shown in Figure 1, where there exists a $\theta^\star$ that results in the optimum point $f^\star = f(\theta^\star)$, and it is defined an estimation error $\tilde{\theta} = \theta^\star - \hat{\theta}$.

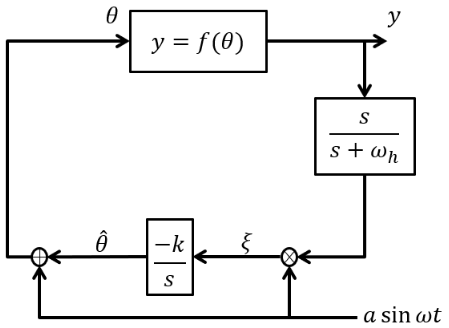

Fig. 1. ESC block diagram.

The function can be approximated around the equilibrium point such that:

$$f(\theta) = f^\star + \frac{f''}{2}(\theta - \theta^\star)^2. \tag{1}$$

From the block diagram in Figure 1 it is observed that $\theta = \hat{\theta} + a\sin(\omega t)$, so

$$\theta - \theta^\star = a\sin(\omega t) - \tilde{\theta}, \tag{2}$$

which, substituting in (1), results in

$$f(\theta) = f^\star + \frac{f''}{2}(\hat{\theta} - a\sin(\omega t))^2, \tag{3}$$

then expanding, results in

$$y = f^\star + \frac{a^2 f''}{4}(1 + \cos(2\omega t)) + \frac{f''}{2}\tilde{\theta}\sin(\omega t). \tag{4}$$

As the high pass filter is designed to cut off all the frequencies below the chosen frequency for ESC, it immediately cuts all constant terms, remaining only the variant ones, resulting in

$$y_{hp} = \frac{f''}{2}\tilde{\theta}^2 - af''\tilde{\theta}\sin(\omega t) + \frac{a^2 f''}{4}\cos(2\omega t). \tag{5}$$

Finally, y_{hp} is multiplied by $a\sin(\omega t)$ and then passed through the integrator. As it is shown in (Ariyur et al., 2003), the average of the output integrator is

$$\dot{\hat{\theta}} \approx \frac{-a^2 k f''}{2}\tilde{\theta}, \tag{6}$$

that, if $kf'' > 0$, it is stable, so $\tilde{\theta} \to 0$, that implies $\hat{\theta} \to \theta^\star$.

This proves the convergence of the method.

3. QUADRUPLE TANK

The quadruple tank is formed by four interconnected tanks. The system has two pumps (controllable inputs) that pour liquid to the tanks coupled by valves (Johansson, 2000; Astrom et al., 2002). A schematic drawing is shown in Figure 2.

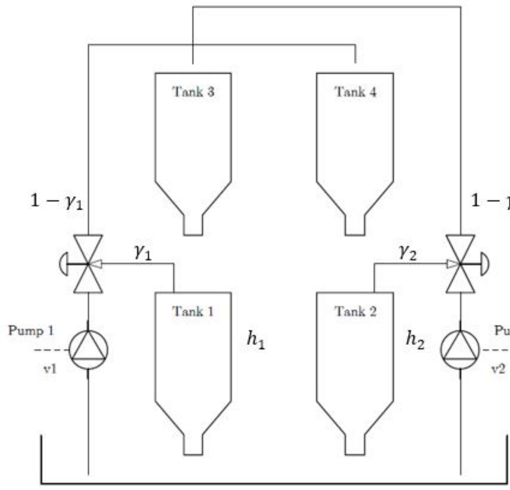

Fig. 2. Schematic of a Quadruple Tank.

The dynamic model is based on the mass balance in each tank (Johansson, 2000). Applying mass conservation to each tank, it is obtained the nonlinear model

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1 v_1}{A_1}, \tag{7}$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2 v_2}{A_2}, \tag{8}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2 v_2}{A_3}, \tag{9}$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1 v_1}{A_4}, \tag{10}$$

where k_i and v_i are the gain and the input voltages, h_i, a_i and A_i are the liquid level, the output hole area and the section area of the tank i, respectively. Besides that, γ_i represents the percentage opening of the i-th valve.

The considered tank has the following constants:

- $g = 981$ cm/s^2;
- $k_1 = k_2 = 5.0926$ cm^3/s;
- $\gamma_1 = \gamma_2 = 0.6$;
- $a_1 = a_2 = 0.3167$ cm^2;
- $a_3 = a_4 = 0.1781$ cm^2;
- $A_1 = A_2 = 28.2743$ cm^2;
- $A_3 = A_4 = 28.2743$ cm^2.

The model is used only for simulation purposes. All controllers are designed without any knowledge of the model.

The quadruple tank system has an interesting feature in relation to its multivariate zero. Depending on the opening of the valves (γ_1 and γ_2) the system may have a minimum phase zero or a non-minimum phase zero (Johansson, 2000). The system will have a minimum phase zero if

$$\gamma_1 + \gamma_2 > 1, \tag{11}$$

and a non-minimum phase zero if

$$\gamma_1 + \gamma_2 < 1. \tag{12}$$

4. CONTROLLER SCHEME DESIGN

This section presents the control approach proposed, considering the ESC proposed in Section 2. For this, it is necessary to guarantee, by hypothesis, that the frequency of the ESC is higher than the natural frequencies of the plant. That is not a very restrictive hypothesis, the idea is to maintain a high frequency so that it is possible to see the oscillations at the output. When using ESC as a direct controller, we want to minimize the quadratic error, which, in most systems, is equal to zero. A block diagram is shown in Figure 3.

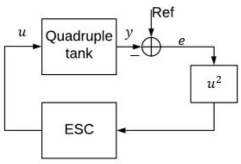

Fig. 3. Direct ESC control block diagram.

The controller is considered decoupled, and the ESCs for each output have the same parameters, shown in Figure 4, where $\phi = 0$ to $i = 1$ and $\phi = \pi/2$ to $i = 2$.

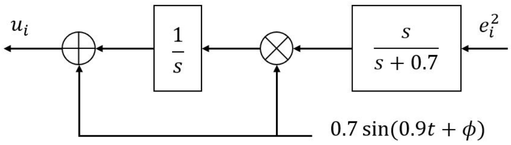

Fig. 4. Direct ESC algorithm by channel.

As this paper uses ESC as a controller that directly actuates on the plant, the constrains of control efforts must be considered. In this case, $0\text{V} \leq u \leq 10\text{V}$.

5. RESULTS

This section presents the simulation results obtained. They are separeted into four cases:

5.1 Case a

In this case, the quadruple tank is opperating at minimum phase ($\gamma_1 = 0.6$ and $\gamma_2 = 0.6$) and it is considered a pulse with an initial condition of $h_1 = 9$ cm and $h_2 = 9$ cm.

Figure 5 shows the simulation result of the system with ESC as controller. The control effort and the quadratic error are shown in Figure 6. The system presented a good result, so that the tanks 1 and 2 converged to the reference. But, the control effort of this approach is very oscillatory. Note that the control effort presents the sinusoidal variation from the ESC algorithm.

5.2 Case b

The second case considers the same opening valve than the case before, but now it is used a sinusoidal reference for each tank with an initial condition of $h_1 = 12$ cm and $h_2 = 12$ cm.

The results obtained in Figures 7 and 8 from the non-model based control were satisfactory. Tank 2 had a slightly larger error and with a greater deviation in the initial condition when compared to tank 1.

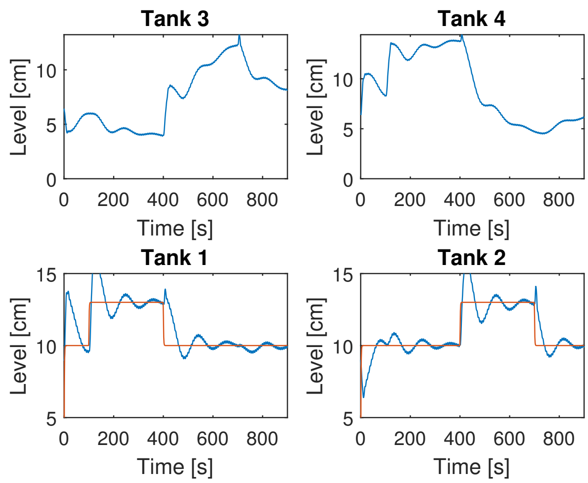

Fig. 5. Outputs considering a pulse reference.

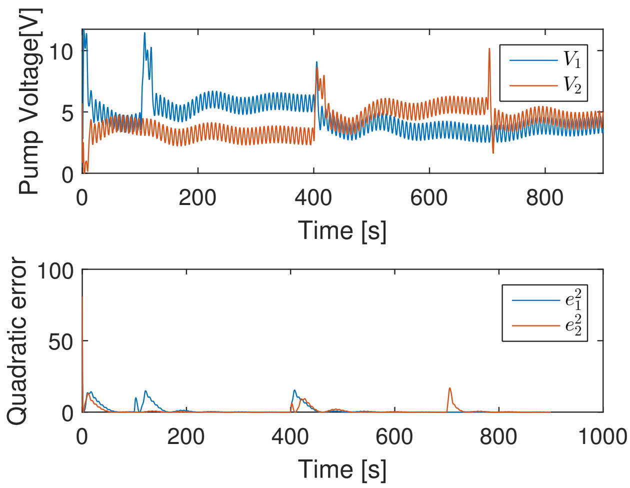

Fig. 6. Control effort and quadratic error considering a pulse reference.

5.3 Case c

The case c represents the same situation of the case a (reference and initial condition), but now we are considering that γ_1 and γ_2 are time variants, as shown in Figure 9. Note that, in this situation, the system spends a short period in non-minimum phase.

Figure 10 shows the output system and Figure 11 shows the control effort and quadratic error. In this situation, considering γ_1 and γ_2 time variants, we obtained a more oscillatory result than the result obtained in case a. But it still has a good performance, as the variation of γ_i causes a big change in the system (including the switch to non-minimum phase).

5.4 Case d

Finally, the case d is the combination of case b with case c. The reference is the same considered in the case b, but now with γ_1 and γ_2 time variants, as shown in Figure 9.

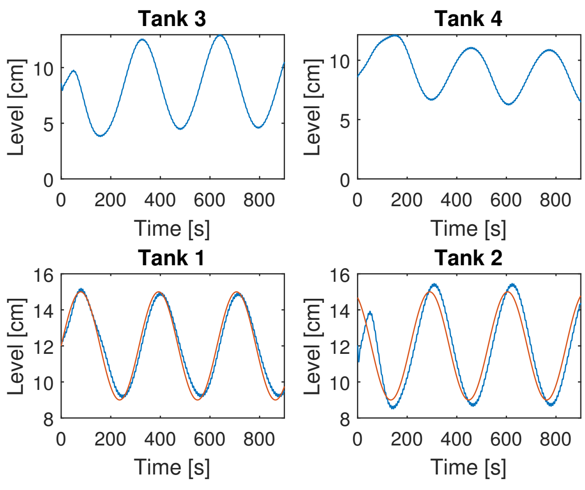

Fig. 7. Outputs considering a sinusoidal reference.

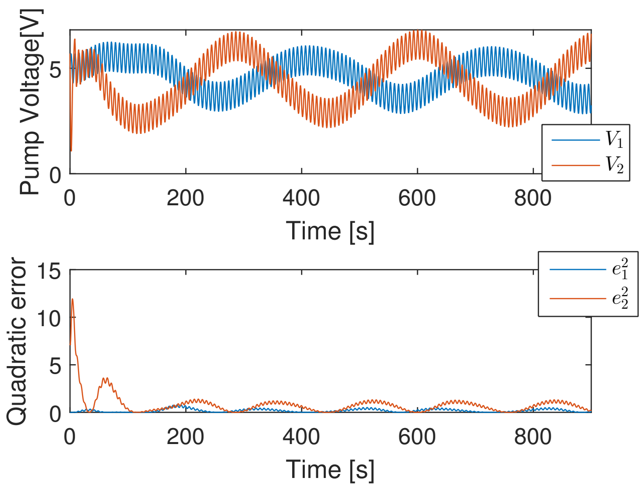

Fig. 8. Control effort and quadratic error considering a sinusoidal reference.

Figures 12 and 13 show the output variables and the control effort with quadratic error, respectively. The result obtained is similar to the case c. The output followed the reference but with a small oscillation around it.

6. CONCLUSION

This paper presents an extremum-seeking control used to solve a tracking problem in a quadruple tank system. The ESC project does not consider the model of the system, generating a model-free control technique.

The control technique is composed of two decentralized ESC algorithms, that is, the first output is associated with the first input and the same is done for the second output.

To test the technique, some simulations were made using the nonlinear quadruple tank model. The results obtained were satisfactory, validating the model-free control technique. Furthermore, the performance was verified considering time-varying parameters.

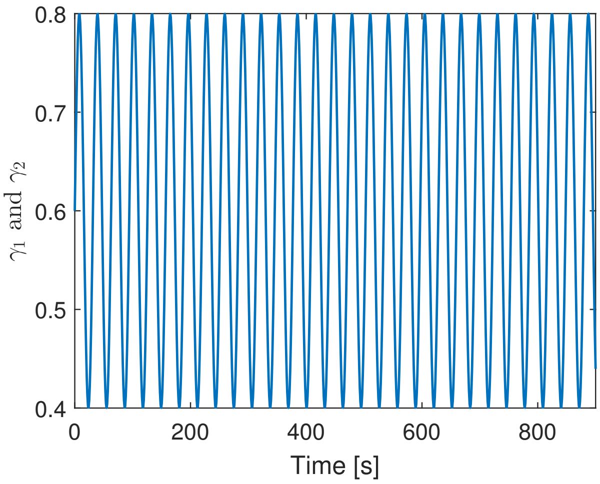

Fig. 9. Time variant γ_1 and γ_2 considered in cases c and d.

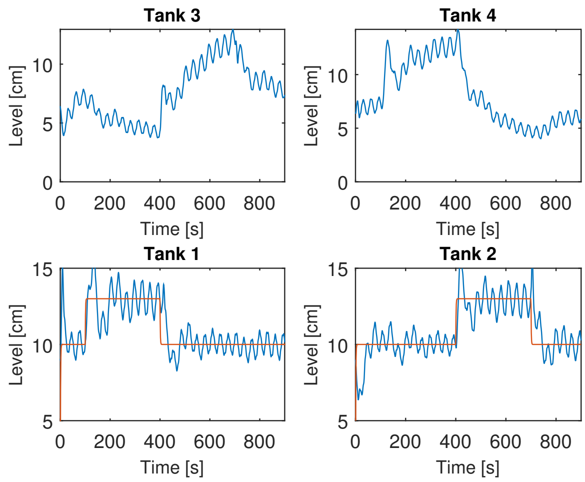

Fig. 10. Outputs considering a step reference and γ_1 and γ_2 time variants.

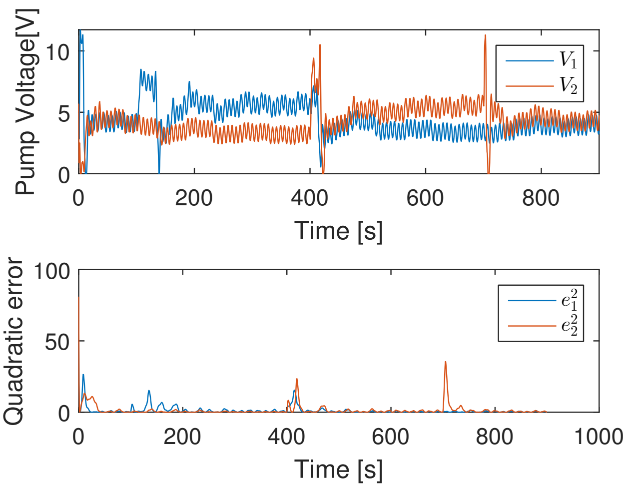

Fig. 11. Control effort and quadratic error considering a step reference and γ_1 and γ_2 time variants.

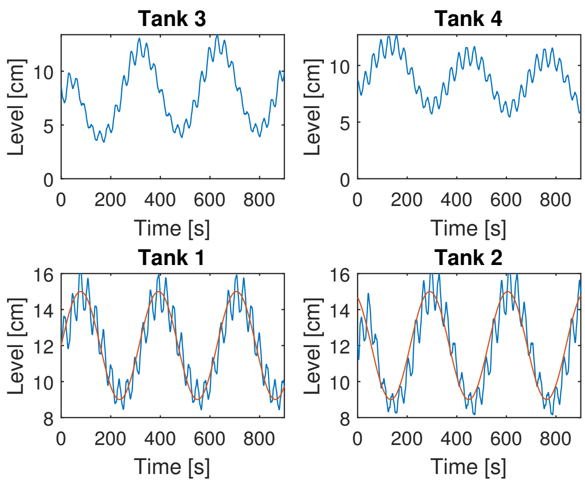

Fig. 12. Outputs considering a sinusoidal reference and γ_1 and γ_2 time variants.

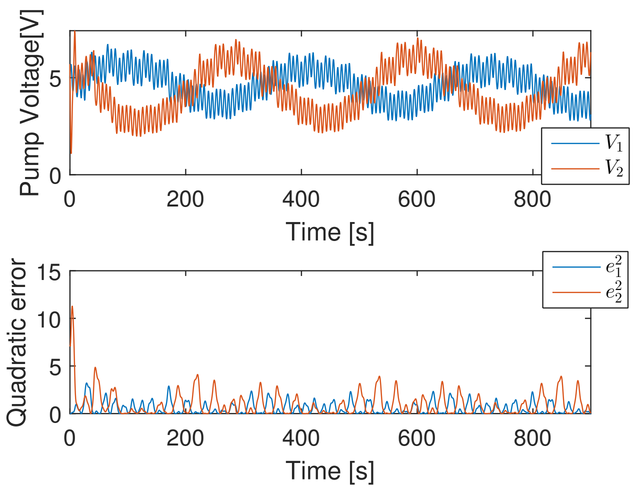

Fig. 13. Control effort and quadratic error considering a sinusoidal reference and γ_1 and γ_2 time variants.

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