Nonlinear Tire Model Approximation Using Artificial Neural Networks ⋆

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Abstract: This paper presents a comparative study between two different approximation approaches for the traditional Magic Formula tire model based on Artificial Neural Network (ANN), in specific, Radial Basis Function Network and Multilayer Perceptron Network, with the view to approximate longitudinal and lateral friction coefficients curves. Simulation results are considered satisfactory, showing that, the MLP network presented better results compared to the RBF network which indicates that the prediction of the friction coefficients curves is driven close to the reference derived from the Magic Formula. A nonlinear physical-mathematical model is used as the real plant for the comparison between the MLP tire model and the Magic Formula, considering torque and different steering angles as input. Moreover, the results show that the implemented network is adequate for applications in simulated ground vehicles.

Keywords: Magic formula, tire model approximation, neural networks.

1. INTRODUCTION

Tires are the components that most contribute to ground vehicle dynamics (Pacejka, 2012). In fact, tires are the link between the vehicle and the environment with the capability to transmit forces that influence vehicle motion. Therefore, the knowledge of friction aspects on the contact patch is a key point for the vehicle safety systems as well as in the improvements of noise emission and fuel economy (Cabrera et al., 2018). Moreover, one of the most important topics of vehicles research has been the design of traction/braking control systems since the loss of adhesion between the tire and the surface leads to vehicle instability.

Different approaches to measure, estimate and predict the friction between the tire and the road have been developed and are still an important topic for research groups (Cheng and Lu, 2017; Cabrera et al., 2018; Olazagoitia et al., 2020). Tire models are frequently used to provide friction coefficients approximation and can classified as physical models, in which the tire is modeled by differential equations, and empirical models based on experimental results Wong (2008). Within the empirical models, one of the most popular models is known as Pacejka's tire model or "Magic Formula", firstly proposed by Bakker et al. (1987), where measured (or simulated) data have to fit the parameters of predefined equations. These equations can be used to predict friction coefficients with certain precision with longitudinal slip and sideslip angles as input (Jazar, 2017; Savaresi and Tanelli, 2010).

Previous work have used tire models in order to predict friction coefficients, such as Lyapunov-based observers to estimate parameters of LuGre friction model (Alvarez et al., 2005; Patel et al., 2008). In Long and Chen (2010), a neural network tire model is applied to predict both longitudinal and lateral forces on the tire by means of the estimation of the Magic Formula parameters. Cabrera et al. (2018) proposed a modification of the original Magic Formula in which effects of road composition, tire type and slippery are included. Cheng and Lu (2017) proposed an interpolation method to identify Magic Formula parameters. Olazagoitia et al. (2020) investigated the use of Artificial Neural Networks (ANNs) to predict Magic Formula parameters. Chen et al. (2018) used online gradient descent algorithm to estimate both sideslip angle and lateral friction coefficient. Bardawil et al. (2020) applied the similarity method on Pacejka's tire model to estimate the friction coefficient in the longitudinal direction.

Previous studies have used different methodologies to approximate tire models as López et al. (2010) which used approximation theory to obtain different types of approximations to the traditional Magic Formula, such as, rational functions, expansions in a series of Chebyshev polynomials, and a series of rational orthogonal functions. Wang et al. (2018) applied a multilayer feed-forward neural network to build an intelligent tire. The peak-value of the tire–road friction curve to control the slip using linear methods is the main topic of the work presented by Satzger et al. (2014); Corno et al. (2009).

In these researches, the friction curves are commonly determined by the prediction of Magic Formula parameters which may lead to high computational effort. Alternatively, friction curves can be approximated by means of ANNs which are a class of models that can find patterns from data. These approaches present characteristics of learning and adaptation to any complex problems with accuracy (Engelbrecht, 2007). One of the main advantages

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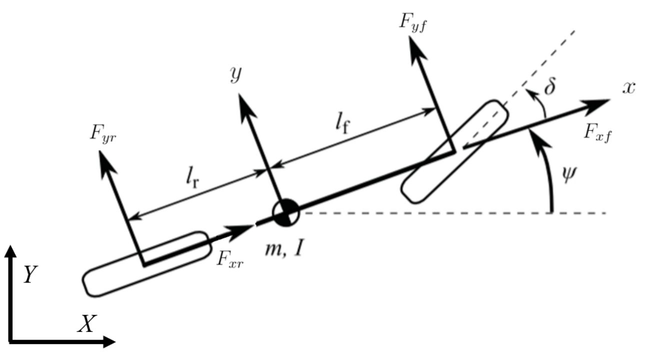

Figure 1. Nonlinear bicycle model used as the real plant.

of these methods is that Neural Network does not require prior information knowledge of the system, which may leads it to be classified as black-box models and is, in fact, a function that interpolates the systems output (Ljung, 2010).

Therefore, the aim of this work is to provide a contribution towards tire models approximation techniques, in specific, ANNs, and their influence in a dynamic vehicle model. This paper presents a comparative study between two different networks, Multilayer Perceptron (MLP) and Radial Based Functions Neural Networks (RBFNNs) architectures in order to obtain an approximate model for Magic Formula.

The remainder of this paper is organized as follows. In section 2, the vehicle model and the lateral and longitudinal mathematical equations of the Magic Formula are presented and discussed. In section 3, MLP and RBFNNs architectures are described. Section 4 exposes the numerical results in terms of synthesis and accuracy. Finally, Section 5 draws the final remarks and point future research directions.

2. VEHICLE DYNAMIC MODEL AND MAGIC FORMULA

The bicycle model (Figure 1) is a simple and functional model that has been used in vehicle dynamics and control (Wang et al., 2019). In this model, the vehicle is assumed to move on a rigid surface with aerodynamic and suspension efforts neglected to simplify the model. Let x and y be the directions (longitudinal and lateral, respectively) in the vehicle frame, X and Y the same directions in the absolute frame, ψ and Ψ the yaw angle in the (x, y) and (X, Y) frames, respectively. The governing equations of motion in the vehicle frame, considering wheels dynamics can be derived as follows:

$$m\ddot{x} = m\dot{y}\dot{\psi} + 2(F_{xf}\cos\delta - F_{yf}\sin\delta) + 2F_{xr}\,, \tag{1a}$$
$$m\ddot{y} = -m\dot{x}\dot{\psi} + 2(F_{xf}\sin\delta + F_{yf}\cos\delta) + 2F_{yr}\,, \tag{1b}$$
$$I\ddot{\psi} = 2l_f(F_{xf}\sin\delta + F_{yf}\cos\delta) - 2l_rF_{yr}\,, \tag{1c}$$
$$I_{\omega f}\dot{\omega}_f = -2F_{xf}r_d + T_t\,, \tag{1d}$$
$$I_{\omega r}\dot{\omega}_r = -2F_{xr}r_d\,, \tag{1e}$$

where m and I are the vehicle mass and the moment of inertia, respectively, l_f and l_r are the front and rear axle distance from the center of gravity (CG), F_{xf} and F_{xr} as well as F_{yf} and F_{yr} are the longitudinal and lateral forces acting on front and rear axles. Besides, $I_{\omega f}$ and $I_{\omega r}$ are the moment of inertia of the front and rear wheels, r_d is wheel radius, T_t is the total torque applied to the driven wheels. The front and rear wheel rotational speed are given by ω_f and ω_r, respectively. Finally, δ is the steering angle.

The vehicle coordinates in the global frame are determined based on the kinematic model given by:

$$\dot{X} = \dot{x}\cos\Psi - \dot{y}\sin\Psi\,, \tag{2a}$$
$$\dot{Y} = \dot{x}\sin\Psi + \dot{y}\cos\Psi\,, \tag{2b}$$
$$\dot{\Psi} = \dot{\psi}\,. \tag{2c}$$

Tire efforts can be determined by the "Magic Formula" which is an important method in vehicle dynamics either in academic or throughout industry to predict the interaction at the contact patch. This model consists of conduct real/virtual tests with the tire to fit the equation parameters to the test data, and consequently, lateral and longitudinal efforts can be determined with accuracy (Blundell and Harty, 2015; Jazar, 2017). In this work, this model is used to determine both longitudinal and lateral friction coefficients at the contact region between the tire and the road. The Magic Formula can be expressed by

$$F(\lambda) = D\sin\{C\mathrm{atan}[B\lambda - E(B\lambda - \mathrm{atan}(B\lambda))]\}\,, \tag{3}$$

where λ coefficient should be replaced by longitudinal slip (s) or sideslip angle (α) for longitudinal and lateral forces (F), respectively. Moreover, sideslip angles for the front (α_f) and rear (α_r) tires can be determined, respectively, by (4) and (5), as follows

$$\alpha_f = \delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\,, \tag{4}$$

$$\alpha_r = -\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\,. \tag{5}$$

On the other hand, the longitudinal slip present on the tire takes the form

$$s = \frac{\dot{x} - \omega r_d}{\max[\dot{x}, \omega r_d]}\,. \tag{6}$$

The other parameters present inside the Magic Formula are expressed by

$$b_x = (a_3Fz^2 + a_4F_z)/(exp(a_5F_z))\,, \tag{7a}$$
$$b_y = a_3\sin(a_4\mathrm{atan}(a_5F_z))\,, \tag{7b}$$
$$B = b_{x,y}/CD\,, \tag{7c}$$
$$C = a_6\,, \tag{7d}$$
$$D = a_1F_z^2 + a_2F_z\,, \tag{7e}$$
$$E = a_7F_z^2 + a_8F_z + a_9\,. \tag{7f}$$

where F_z is the vertical effort at the contact point and b_x, b_y are parameters related to longitudinal and lateral directions, respectively. Moreover, a_p, with $p = 1, 2, ...9$, are coefficients with different values for longitudinal and lateral directions.

It is important to point out that both longitudinal and lateral efforts depend on different features of the road and tire. Thus, they generally can be described as:

$$F_x = F_x(F_z, s), \tag{8}$$

$$F_y = F_y(F_z, \alpha). \tag{9}$$

A normalized expression of the vertical forces, (8) and (9), is typically considered, and therefore the proportionally longitudinal (μ_x) and lateral (μ_y) friction coefficients can be defined as

$$\mu_x(s) := \frac{F_x}{F_z}, \tag{10}$$

$$\mu_y(\alpha) := \frac{F_y}{F_z}. \tag{11}$$

3. ARTIFICIAL NEURAL NETWORK (ANN)

ANN is a mathematical representation inspired in the manner of how the brain performs a particular task. It can train and adapt itself considering different datasets by adjusting its connections and parameters (Haykin, 2009). Despite their particular differences, ANNs have the same pattern: they are composed by simpler elements (neurons) which build complex structure considering a set of inputs to produce a set of approximations as a mapping (Wasserman, 1993). Relevant networks are the multilayer perceptron (MLP), and radial basis functions neural networks (RBFNNs) (Haykin, 2009).

The radial basis function network is a network with a simple architecture since uses radial basis functions as an activation function (Mazhar et al., 2019). Besides, one important characteristic of RBFNNs is the presence of linear-in-the-parameters weighting coefficients resulting in faster learning algorithms (Ayala, 2016). Essentially, RBFNNs are composed of the input, hidden and output layers. The input layer connects the input data with the source nodes without weighting these inputs. On the other hand, the hidden layer is composed of different neurons of the activation functions with their outputs weighted and summed in the output layer, which is the RBFNN final approximation. A RBFNN can be mathematically be written as

$$\underline{\hat{y}}(t) = \sum_{i=1}^{M} \omega_i \phi(r(t), c_i, \sigma_i)\,, \tag{12}$$

where $\underline{\hat{y}}(t)$ is the network identified output, M is the quantity of neurons in the hidden layer, ω_m is the output weights coefficient, $r(t)$ is the input vector, c_i and σ_i are the center and the width of the i-th hidden node, respectively.

Typical choices for activation function are the Gaussian (13a) and the multiquadratic function (13b) which can be determined by

$$\phi_g(l) = exp\left(\frac{l^2}{\sigma_i^2}\right), \tag{13a}$$

$$\phi_m(l) = \sqrt{l^2 + \sigma_i^2}\,, \tag{13b}$$

where l stands for the Euclidean norm between the input of the neural network r to a given center c, i.e. $l = \|r - c\|$

Some parameters of a RBFNN can be adjusted to obtain a better final result, among them, the number of neurons in the hidden layer, the width and the position of the centers of the RBFs, as well as, the output weights. In order to achieve results with accuracy, the RBFNN parameters are defined by means of a supervised method. To this end, in this work, the minimization of the sum of squared errors is assumed and solved by the Genetic Algorithm where the optimization problem is expressed by

$$\text{min} f(x) = < \underline{y}(t) - \underline{\hat{y}}(t) >, \tag{14}$$

$$\hat{x} = [c_1^T, ..., c_i^T, \sigma_1, ..., \sigma_i, \omega_1, ..., \omega_i]^T, \tag{15}$$

where $\underline{y}(t)$ is the vector of observations until time t, namely

$$\underline{y}(t) = \begin{bmatrix} \underline{y}(1) \\ \underline{y}(2) \\ \vdots \\ \underline{y}(n) \end{bmatrix}. \tag{16}$$

The vector of decision variables $\hat{x}$ contains the RBFNN parameters to be derived from the supervised fashion.

Another important type of network is the Multilayer Perceptron network which was proposed to solve nonlinearly separable problems. Some features include: differentiable activation functions, presence of one or many hidden layers, high degree of connectivity. Mathematically, MLP architecture is more complex when compared to the RBFNN, and therefore, this network only deemed viable when researchers found a way to train these architectures by means of the backpropagation algorithm (Haykin, 2009). Considering a MLP network having two layers of weights and linear output units with a set of inputs x_i and outputs y_k. The output of the MLP network takes the form:

$$\underline{\hat{y}}_j = f\left(\sum_{i=1}^{Z} \omega_{ij} x_{ij} + \omega j\right), \tag{17}$$

where $\underline{\hat{y}}_j(t)$ is the network identified output, Z is the number of hidden layers, ω_{ij} are the weights coefficient between the i-th neuron, in the prior layer, and the j-th neuron in the actual layer, ω_j is the bias weight, i is the number of neurons that have been connected to the j-th neuron, while x_{ij} are the input signals from the i-th neuron to the j-th neuron.

In the present work, we use a MLP network composed by two hidden layers with nodes totally connected by means of a Feed-Forward method, activated through the sigmoid ("tanh") function that allows training through the Backpropagation Algorithm (Bishop, 2006).

4. NUMERICAL RESULTS

In this section, both MLP and RBF networks are compared with the traditional Pacejka's tire model for longitudinal and lateral friction approximation. The datasets containing 200 data points are generated by calculating the friction coefficients μ_x and μ_y using Pacejka's tire model by means of the parameters listed in Table 1, which are based on a military vehicle with a weight distribution 50/50 .

Table 1. Vehicle Parameters

Parameter	Description	Value	Unit
m	Vehicle mass	2500	kg
I	Moment of inertia of the vehicle	2200	kg.m^2
$I_{\omega f/\omega r}$	Moment of inertia of the wheel	2.5	kg.m^2
l_t	Wheelbase	2.7	m
r	Wheel radius	0.42	m

The coefficients a_p for the Pacejka's model were obtained from the work developed by Bakker et al. (1987) and are listed in Table 2.

Table 2. Coefficients from Pacejka model (Bakker et al., 1987)

μ_x		μ_y	
Parameter	Value	Parameter	Value
a_1	-21.3	a_1	-22.1
a_2	1144	a_2	1011
a_3	49.6	a_3	1078
a_4	226	a_4	1.82
a_5	0.069	a_5	0.208
a_6	1.65	a_6	1.30
a_7	-0.006	a_7	0
a_8	0.056	a_8	-0.354
a_8	0.486	a_8	0.707

In order to compare the simulation results, two error-based metrics are considered in this work: Root Mean Squared Error - RMSE (in decibels) and Multiple Correlation Coefficient (R^2). These metrics are formulated as

$$\text{RMSE} = 10 \log_{10} \sqrt{\frac{1}{N}\sum_{t=1}^{N}[\underline{y}(t) - \underline{\hat{y}}(t)]^2}\,, \tag{18}$$

$$\text{R}^2 = 1 - \frac{\sum_{t=1}^{N}[\underline{y}(t) - \underline{\hat{y}}(t)]^2}{\sum_{t=1}^{N}[\underline{y}(t) - \underline{y}_{mean}(t)]^2}\,, \tag{19}$$

where $\underline{y}$ is the reference input, $\underline{\hat{y}}$ is the approximated data, $\underline{y}_{mean}$ is the mean value of the input signal, and N is the length of the set.

4.1 Neural Network Results

Gaussian and multiquadratic functions are used as activation functions for RBF networks, considering 2, 3, and 4 neurons in the hidden layer. Besides, in order to determine the spread and the centers, the sum of squared errors is applied as the cost function to be minimized for the GA algorithm. In this case, only the centers are limited by lower and upper bounds [-1;1].

The results for the longitudinal friction (μ_x) and lateral friction (μ_y) are shown in Figure 2 and 3. It can be

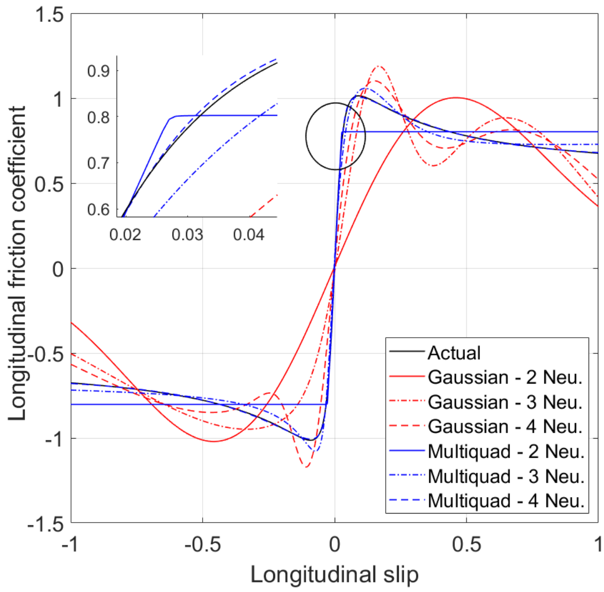

Figure 2. Longitudinal friction approximation using RBF network for different activation functions and number of neurons in the hidden layer.

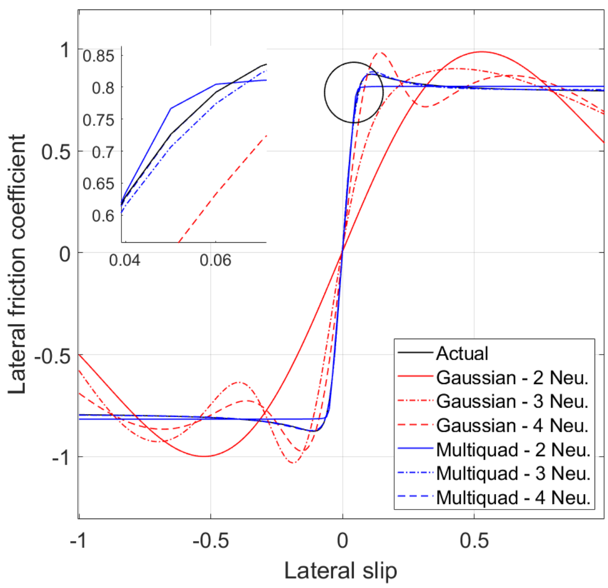

Figure 3. Lateral friction approximation using RBF network for different activation functions and number of neurons in the hidden layer.

seen that the gaussian function does not perform well if compared with the multiquadratic function even for a large number of neurons. The results near to zero slip are satisfactory for gaussian function, but far from the origin, the responses oscillate around the actual (ideal) curve, specifically 2 and 3 neurons. On the other hand, the multiquadratic function presents a satisfactory result when 3 and 4 neurons are considered. In particular for lateral friction, the curve is smoother than the longitudinal curve, and therefore, the results from 3 and 4 neurons are closer to the ideal.

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The results can be corroborated by the metric results (Table 3), in which the RBF network with 4 neurons in the hidden layer that considers the multiquadratic function presents the best results for both RMSE (dB) and R^2.

Table 3. RBF results for longitudinal and lateral friction coefficients.

Parameter	Metric	Activation Function	Neurons 2	3	4
μ_x	RMSE	Multiquad	-9.88	-14.35	-21.98
		Gaussian	-5.22	-7.63	-10.12
	R^2	Multiquad	0.98	0.98	0.99
		Gaussian	0.85	0.95	0.98
μ_y	RMSE	Multiquad	-16.29	-23.34	-28.11
		Gaussian	-6.11	-8.85	-11.44
	R^2	Multiquad	0.99	1.00	1.00
		Gaussian	0.91	0.97	0.99

For the MLP network, two hidden layers with 1, 2, and 3 neurons in each hidden layer are used to the comparison. The metric results for both longitudinal and lateral friction coefficients are listed in Table 4. It can be noted that the results are closer to the ideal considering (R^2 metric), if compared to the RBF network, even when one neuron in each hidden layer is considered.

Table 4. MLP results for longitudinal and lateral friction coefficients.

Friction Coefficient	Metric	[1,1] Neurons	[2,2] Neurons	[3,3] Neurons
longitudinal	RMSE (dB)	-9.685	-20.294	-19.809
	R^2	0.982	0.999	0.998
lateral	RMSE (dB)	-15.615	-22.996	-24.426
	R^2	0.998	1.00	1.00

Thus, the MLP network with two neurons in each hidden layer yields the best result for longitudinal and lateral friction coefficients considering all the metrics evaluated. From Figure 4, we can see the results for the best MLP case.

4.2 Dynamic Tests

At this section, we perform vehicle dynamic virtual tests considering the best neural network tire model obtained in the previous analysis, which was the MLP network with two hidden layers and two neurons in each layer.

A constant torque equals to 1200 N.m is applied to the driven wheels, as well as, different steering angles are used as input for the nonlinear bicycle model (1). Three comparative tests considering sinusoidal trajectories are performed to investigate the comparison between pacejka and MLP tire models. Curved trajectories allow the visualization of discrepancies due to the presence of nonlinearities in vehicle dynamics, including tire models.

In the first test (Test 1), a steer angle based on a sinusoidal function (amplitude of -0.05 m and angular frequency of 0.4 rad/s) is given to the steering wheel. From Figure 5 we can see that the performance of the dynamic model with MLP tire model is satisfactory compared with the Pacejka tire model. Besides, some deviation can be noted through the simulation representing a lateral offset mean error of 0.10 m. However, these discrepancies are not sufficient to impact the final result.

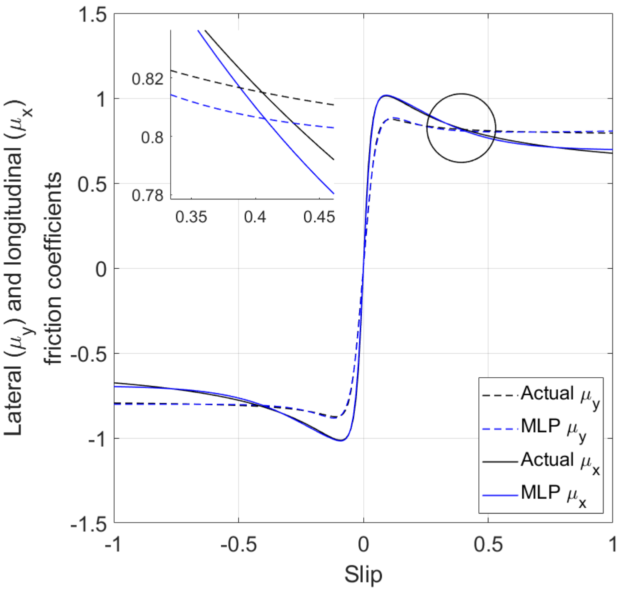

Figure 4. Longitudinal and lateral friction approximation using MLP network for different number of neurons in the hidden layer.

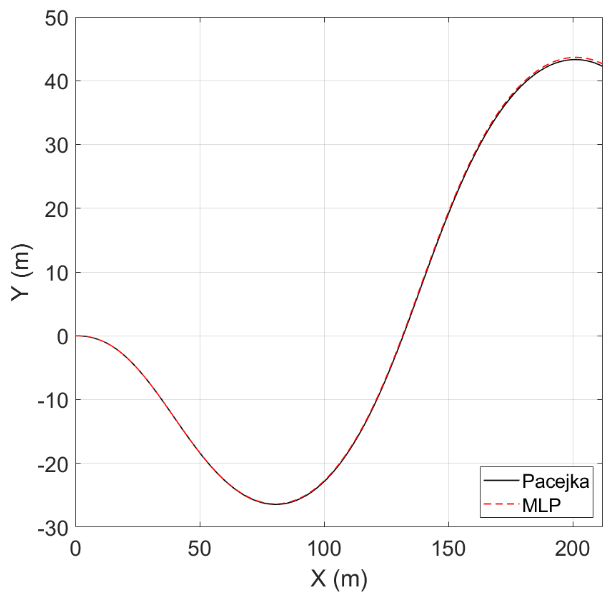

Figure 5. Trajectory comparison between vehicle dynamic model with Pacejka and MLP network tire models - Test 1.

In second test (Test 2), a steering angle based on a sinusoidal function (amplitude of 0.08 m and angular frequency of 0.4 rad/s), is given to the steering wheel. Here, we can see that the trajectory response obtained from the MLP network (Figure 6) is close to the Pacejka tire model. In this case, a lateral offset mean error of 0.12 m is measured through the simulation. This is due to the high amplitude of the motion compared to the previous test.

In the latter test (Test 3), a steering angle based on a sinusoidal function (amplitude of 0.08 m and angular frequency of 0.4 rad/s) followed by a zero angle, is given

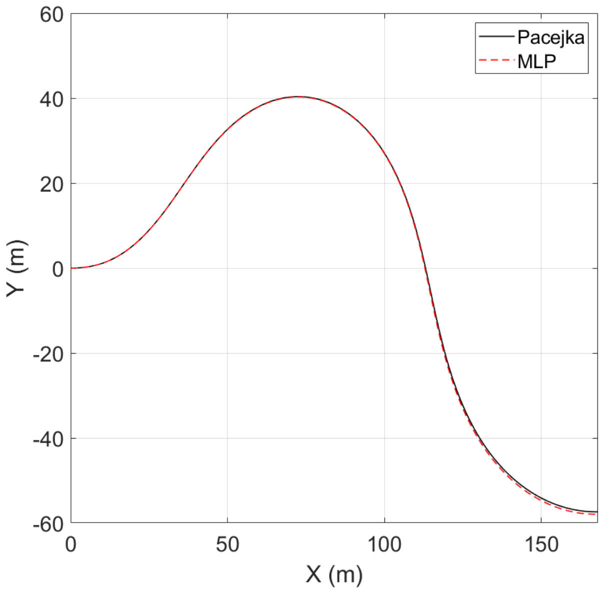

Figure 6. Trajectory comparison between vehicle dynamic model with Pacejka and MLP network tire models - Test 2.

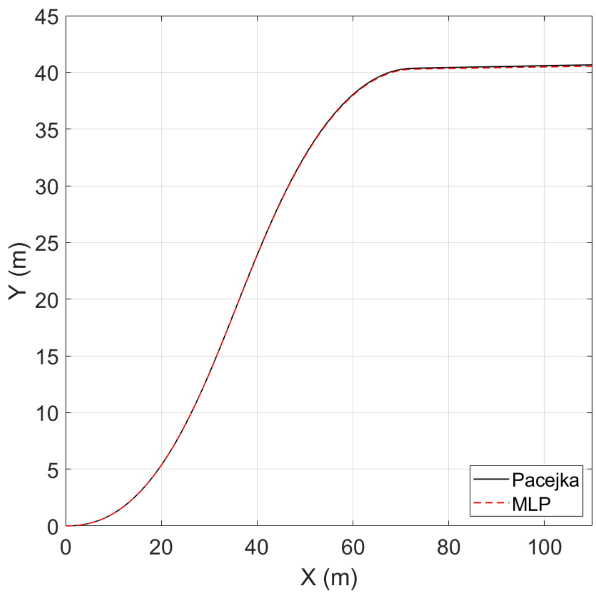

Figure 7. Trajectory comparison between vehicle dynamic model with Pacejka and MLP network tire models - Test 3.

to the steering wheel. From Figure 7, we also can see that the MLP response achieve satisfactory performance. The straight direction, after 60 m (in longitudinal direction), maintains constant the lateral offset mean error through the simulation. In this case, the error value is equal to 0,02 m.

5. CONCLUSIONS

The present work dealt with the application of artificial neural network to approximate the well-known "Magic Formula" tire model since numerous difficulties in modeling, control, and simulation of vehicle emerge due to the complexity related to tire and contact soil properties which may lead to a high difference between physical and virtual models.

Multilayer Perceptron and radial basis functions networks are compared by means of different number of neurons and activation functions. Based on the results, we conclude that the MLP network performed better for all tests when compared with RBF network. However, it is important to point out that the RBF network, with the multiquadratic activation function, also presented satisfactory results when three or more neurons are considered.

A bicycle dynamic model was derived to test the MLP network. The results, supported by the metric results, demonstrate the potential of ANNs in identification and estimation of vehicle dynamics characteristics.

In the future we shall focus on implement a trajectory tracking control by means of a model predictive control (MPC); improve the dynamic model adding a more complex tire model for off-road situations as presented by Taghavifar and Rakheja (2020).

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