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Safe Control of a 2DOF Helicopter Using Exponential Control Barrier Function ⋆

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Abstract: This work presents the safe control of a 2 degrees-of-freedom (2DOF) helicopter. We consider a control framework that unifies stability/tracking objectives, expressed as a nominal control law, and safety constraints, expressed as control barrier functions (CBFs), through a quadratic programming (QP) and the safety constraints must be prioritized. A linear quadratic regulator (LQR) is applied as the nominal control law and two safety constraints are considered to ensure that pitch and yaw angles never exceed predetermined bounds. As these safety constraints have high relative-degree, we represent them as exponential control barrier functions (ECBFs). The system model is based on a 2DOF helicopter available at EPUSP. The results obtained through numerical simulations demonstrate that the safety constraints are always satisfied and the tracking objectives are satisfied just when are not in conflict with the safety constraints.

Keywords: Control Barrier Function, Safety, 2DOF Helicopter, Quadratic Programming, Optimal Control.

1. INTRODUCTION

The 2 degrees-of-freedom (2DOF) helicopter is a nonlinear and coupled multiple-input-multiple-output (MIMO) system. Several control strategies presented in the literature have been applied to this system, such as robust optimal control using linear matrix inequalities with Takagi-Sugeno fuzzy controllers (Yu (2007)), adaptive robust linear quadratic control (Watanabe et al. (2013)) and nonlinear predictive control (Dutka et al. (2003)). These works are proposed to satisfy tracking objectives, however safety is not considered. Safety is represented by constraints in system states or outputs and can be mathematically related to control barrier functions (CBFs). Thus, we apply a control framework that simultaneously satisfy tracking objectives and safety constraints.

The control framework considered in this work is described in Ames et al. (2014b). More complete and detailed versions of Ames et al. (2014b) can be seen in Ames et al. (2019) and Ames et al. (2017). This control framework unifies stability/tracking objectives, expressed as a control Lyapunov function (CLF) or a nominal control law, and safety constraints, expressed as a CBF. These objectives can be integrated through quadratic programming (QP) and safety constraints must be prioritized. Several applications using this methodology are described in the literature such as bipedal walking robot (Nguyen and Screenath (2015)), robotic manipulator (Rauscher et al. (2016)), two-wheeled human transporter (*Segway*) (Gurriet et al. (2018)), quadrotors (Wu and Sreenath (2016)) and multi-robot systems (Wang et al. (2017)). Besides that, this control framework is only applicable for relative-degree one safety constraints, i.e., the first time-derivative of the CBF has to depend on the control input. However, in several cases, the relative-degree of the safety constraint is greater than one. Some works presented in the literature propose solutions to deal with high relative-degree safety constraints (greater than one) for this control framework. In Wu and Screenath (2015), a solution applied only for relative-degree two safety constraints is proposed. In Hsu et al. (2015), a backstepping-based method is applied to high relative-degree safety constraints. In Nguyen and Sreenath (2016), the concept of exponential control barrier function (ECBF) is introduced as a way to systematically enforce high relative-degree safety constraints.

In this work, we apply the control framework described above for a 2DOF helicopter. The system model is based on a 2DOF helicopter available at Escola Politécnica da Universidade de São Paulo (EPUSP). A linear quadratic regulator (LQR) is applied as the nominal control law to ensure that pitch and yaw angles track reference inputs (tracking objectives) and two safety constraints are considered to ensure that pitch and yaw angles never exceed predetermined bounds. As these safety constraints have high relative-degree, we represent them as ECBFs. The initial results were obtained through numerical simulations. It is important to highlight that this same 2DOF helicopter was controlled in Neto et al. (2016), Neto et al. (2017) and Barbosa et al. (2016) using a pole-placement, a state feedback decoupling control and a discrete linear quadratic Gaussian/loop transfer recovery control augmented by integrators, respectively.

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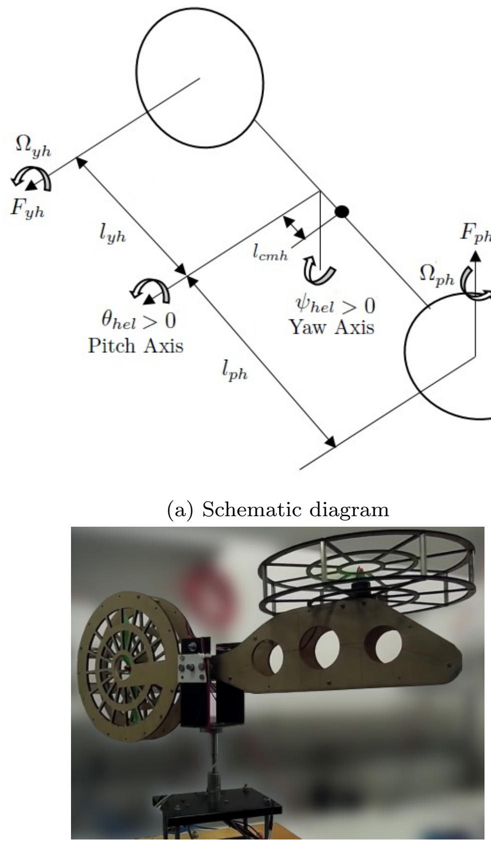
The rest of this paper is organized as follows: In Section 2, the modeling of the 2DOF helicopter is described. The LQR, the concepts of CBF and ECBF, and the control framework that unifies the stability/tracking objectives and the safety constraints through QP are presented in Section 3. The numerical simulations and the conclusions are presented in Sections 4 and 5 respectively.

2. SYSTEM MODELING

The schematic diagram and the prototype of the 2DOF helicopter available at EPUSP are shown in Fig. 1.



(a) Schematic diagram

(b) Prototype

Fig. 1. 2DOF helicopter available at EPUSP

The system rotates around pitch and yaw axes (with angles θ_{hel} and ψ_{hel}) and we consider the following conventions (Barbosa et al. (2016)):

(1) Helicopter is horizontal with $\theta_{hel} = 0$;
(2) $\dot{\theta}_{hel}$ is positive when the nose is moving upwards;
(3) $\dot{\psi}_{hel}$ is positive when the helicopter rotates clockwise;
(4) There is neither rolling nor axial movements.

The system is composed by two brushless DC motors with propellers that control pitch and yaw angles applying forces F_{ph} and F_{yh}. The 2DOF helicopter is a coupled system, i.e., the pitch motor controls directly the pitch angle using the force generated by its propeller, creating a torque in yaw as effect of air resistance. Thus, the pitch and yaw resulting torques are written as (Barbosa et al. (2016)):

$$\tau_{ph} = K_{pp}V_{mp} + K_{py}V_{my}, \tag{1}$$

$$\tau_{yh} = K_{yy}V_{my} + K_{yp}V_{mp}, \tag{2}$$

where K_{pp} is the pitch motor thrust constant, K_{py} is the yaw motor torque constant, K_{yy} is the yaw motor thrust constant, K_{yp} is the pitch motor torque constant, V_{mp} is the pitch motor percentage, V_{my} is the yaw motor percentage, in which this percentage is related to the signal sent to the motor controller.

Applying the Euler-Lagrange formulation, we can obtain the following equations of motion (Barbosa et al. (2016)):

$$\ddot{\theta}_{hel} = \frac{\lambda_{hel} - (B_{ph}\dot{\theta}_{hel} + \alpha_{hel} + \beta_{hel})}{J_{eqp} + m_h l_{cmh}^2}, \tag{3}$$

$$\lambda_{hel} = K_{pp}V_{mp} + K_{py}V_{my}, \tag{4}$$

$$\alpha_{hel} = m_h l_{cmh}^2 \dot{\psi}_{hel}^2 \sin(\theta_{hel})\cos(\theta_{hel}), \tag{5}$$

$$\beta_{hel} = m_h g \cos(\theta_{hel}) l_{cmh}, \tag{6}$$

$$\ddot{\psi}_{hel} = \frac{\rho_{hel} + \gamma_{hel} - B_{yh}\dot{\psi}_{hel}}{J_{eqy} + m_h \cos(\theta_{hel})^2 l_{cmh}^2}, \tag{7}$$

$$\rho_{hel} = K_{yy}V_{my} + K_{yp}V_{mp}, \tag{8}$$

$$\gamma_{hel} = 2m_h l_{cmh}^2 \sin(\theta_{hel})\cos(\theta_{hel})\dot{\psi}_{hel}\dot{\theta}_{hel}, \tag{9}$$

where m_h is the helicopter total moving mass, l_{cmh} is the center of mass distance to origin, B_{ph} and B_{yh} are the movement resistance acting above pitch and yaw axes, respectively, J_{eqp} and J_{eqy} are the equivalent moments of inertia related to pitch and yaw axes, respectively, and g is the gravitational acceleration constant (Neto et al. (2017)).

3. CONTROL FRAMEWORK

This section presents the LQR, the concepts of CBF and ECBF, and the control framework that unifies the stability/tracking objectives and the safety constraints through QP.

3.1 Nominal Control Law - LQR

As described previously, a LQR is applied as the nominal control law $u_{no_{hel}}$ to ensure that pitch and yaw angles track reference inputs (tracking objectives). To design the LQR, we linearize the system for small angles, as this is the operating point. Thus, the equations of motion are defined by:

$$\ddot{\theta}_{hel} = \frac{\lambda_{hel} - B_{ph}\dot{\theta}_{hel} - m_h g l_{cmh}}{J_{eqp} + m_h l_{cmh}^2}, \tag{10}$$

$$\ddot{\psi}_{hel} = \frac{\rho_{hel} - B_{yh}\dot{\psi}_{hel}}{J_{eqy} + m_h l_{cmh}^2}. \tag{11}$$

The system can be described in state-space such as:

$$\dot{x}_{hel} = A_{hel}x_{hel} + B_{hel}u_{hel}, \tag{12}$$

$$y_{hel} = C_{hel}x_{hel}, \tag{13}$$

where $x_{hel} = \begin{bmatrix}\theta_{hel} & \psi_{hel} & \dot{\theta}_{hel} & \dot{\psi}_{hel}\end{bmatrix}^T$ are the states, $u_{hel} = [V_{mp} \;\; V_{my}]^T$ are the inputs, A_{hel} is the state matrix, B_{hel} is the input matrix and C_{hel} is the output matrix. The matrices A_{hel}, B_{hel} and C_{hel} are given by:

$$A_{hel} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{-B_{ph}}{J_{eqp} + m_h l_{cmh}^2} & 0 \\ 0 & 0 & 0 & \dfrac{-B_{yh}}{J_{eqy} + m_h l_{cmh}^2} \end{bmatrix}, \tag{14}$$

$$B_{hel} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{K_{pp}}{J_{eqp} + m_h l_{cmh}^2} & \dfrac{K_{py}}{J_{eqp} + m_h l_{cmh}^2} \\ \dfrac{K_{yp}}{J_{eqy} + m_h l_{cmh}^2} & \dfrac{K_{yy}}{J_{eqy} + m_h l_{cmh}^2} \end{bmatrix}, \quad (15)$$

$$C_{hel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (16)$$

LQR is an optimal regulator that, given the linearized system (12)-(13), determines the matrix K of the optimal control vector

$$u_{no_{hel}} = -Kx_{hel} \quad (17)$$

so as to minimize the performance index

$$J = \int_0^\infty \left(x_{hel}^T Q x_{hel} + u_{no_{hel}}^T R u_{no_{hel}} \right) \mathrm{d}t, \quad (18)$$

where Q is a positive-semidefinite matrix and R is a positive-definite matrix. These matrices are selected to weight the relative importance of the state vector x_{hel} and the input vector $u_{no_{hel}}$ on the performance index minimization (Ogata (2009)).

If there exists a positive-definite matrix P satisfying the Riccati equation

$$A_{hel}^T P + P A_{hel} - P B_{hel} R^{-1} B_{hel}^T P + Q = 0, \quad (19)$$

then the closed-loop system is stable. Thus, the optimal matrix K can be obtained by

$$K = R^{-1} B_{hel}^T P. \quad (20)$$

As we consider the tracking objectives, the control vector (17) is given by:

$$u_{no_{hel}} = -K(x_{hel} - x_{hel_r}), \quad (21)$$

where $x_{hel_r} = \begin{bmatrix} \theta_{hel_r} & \psi_{hel_r} & \dot{\theta}_{hel_r} & \dot{\psi}_{hel_r} \end{bmatrix}^T$ are the reference inputs.

3.2 Control Barrier Function

Initially, we consider the system model represented by:

$$\dot{x} = f(x) + g(x)u, \quad (22)$$

with states $x \in \mathcal{D} \subset \mathbb{R}^n$, inputs $u \in \mathcal{U} \subset \mathbb{R}^m$ and $f(x)$ and $g(x)$ locally Lipschitz.

Safety is verified in terms of a set invariance, i.e., not leaving a safe set. We consider a set $\mathcal{C}$ defined as the superlevel set of a continuously differentiable function $h(x) : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}$ as (Ames et al. (2019)):

$$\begin{aligned} \mathcal{C} &= \{x \in \mathcal{D} \subset \mathbb{R}^n : h(x) \geq 0\}, \\ \partial\mathcal{C} &= \{x \in \mathcal{D} \subset \mathbb{R}^n : h(x) = 0\}, \\ \mathrm{Int}(\mathcal{C}) &= \{x \in \mathcal{D} \subset \mathbb{R}^n : h(x) > 0\}. \end{aligned} \quad (23)$$

Thus, safety can be defined (Ames et al. (2019)):

Definition 1. (Safety) Let u be a feedback controller such that (22) is locally Lipschitz. For any initial condition $x_0 \in \mathcal{D}$ there exists a maximum interval of existence $I(x_0)$ such that $x(t)$ is the unique solution to (22) on $I(x_0)$. The set $\mathcal{C}$ is forward invariant if for every $x_0 \in \mathcal{C}$, $x(t) \in \mathcal{C}$ for $x(0) = x_0$ and $\forall t \in I(x_0)$. The system (22) is safe with respect to the set $\mathcal{C}$ if the set $\mathcal{C}$ is forward invariant.

After the set $\mathcal{C}$ and safety have been defined, the CBF $h(x)$ can formally be defined (Ames et al. (2019)), (Ames et al. (2017)):

Definition 2. (CBF) Consider the control system (22) and the set $\mathcal{C}$ defined by (23) for a continuously differentiable function $h(x) : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}$. The function $h(x)$ is called a CBF defined on set $\mathcal{D}$, if there exists an extended class κ function α such that

$$\sup_{u \in \mathcal{U}} \left[L_f h(x) + L_g h(x) u + \alpha(h(x)) \right] \geq 0, \; \forall x \in \mathcal{D}, \quad (24)$$

where $L_f h = \nabla h(x) \cdot f(x)$, $L_g h = \nabla h(x) \cdot g(x)$ and the class κ function is defined by (Ames et al. (2017)):

Definition 3. (Class κ function) A continuous function $\alpha : [0, a) \to [0, \infty)$ for some $a > 0$ is said to belong to a class κ if it is strictly increasing and $\alpha(0) = 0$.

Considering a CBF $h(x)$, $\forall x \in \mathcal{D}$, define the set (Ames et al. (2019)), (Ames et al. (2017)):

$$K_{cbf}(x) = \{u \in \mathcal{U} : L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0\}. \quad (25)$$

Considering control values in this set, the forward invariance of $\mathcal{C}$ is guaranteed by the following corollary (Ames et al. (2017)):

Corollary 1. Assume the set $\mathcal{C}$ defined by (23) and let $h(x)$ be an associated CBF for the system (22), then any locally Lipschitz continuous controller $u : \mathcal{D} \to \mathcal{U}$ such that $u(x) \in K_{cbf}(x)$ will render the set $\mathcal{C}$ forward invariant.

The final control framework unifies stability/tracking objectives, expressed as a nominal control law u_{no}, and safety constraints, expressed as a CBF, through QP and the safety constraints must be prioritized. Fig. 2 presents a synthesized description of the control framework.

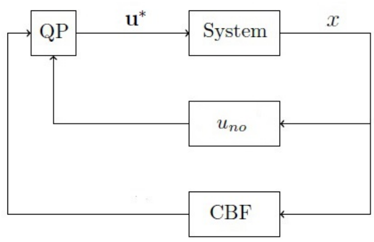


Fig. 2. Synthesized description of the control framework

The controller is formulated as an optimization problem, minimizing the error (Rauscher et al. (2016))

$$e_u = u_{no} - u. \quad (26)$$

The squared norm of the error

$$\|e_u\|^2 = u^T u - 2u_{no}^T u + u_{no}^T u_{no} \quad (27)$$

is considered as the objective function. The last term of (27) is neglected, as it is constant in a minimization with respect to u (Rauscher et al. (2016)). Thus, we can consider the following QP-based controller (Ames et al. (2019)), (Rauscher et al. (2016)):

$$\begin{aligned} \mathbf{u}^*(x) = \; & \underset{u \in \mathbb{R}^m}{\arg\min} \; u^T u - 2u_{no}^T u \\ s.t. \; & L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0. \end{aligned} \quad (28)$$

It is important to highlight that the constraint in QP enforces the condition (24) for the CBF.

The QP-based controller (28) is only applicable for relative-degree one safety constraints, i.e., the first time-derivative of the CBF has to depend on the control input.

When high relative-degree safety constraints are considered $L_g h(x) = 0$ and the QP cannot be solved; thus, we introduce the concept of ECBF to deal with high relative-degree safety constraints.

3.3 Exponential Control Barrier Function

The concept of ECBF was first introduced in Nguyen and Sreenath (2016), where in the final control framework, stability/tracking objectives are expressed as a CLF. However, in this work, the formulation of Nguyen and Sreenath (2016) is adapted in order to express stability/tracking objectives as a nominal control law, such as in (28). The term ECBF is used since the resulting CBF constraint is an exponential function of the initial condition (Nguyen and Sreenath (2016)).

In Ames et al. (2014a), it is described a systematic procedure using input-output linearization to design CLFs for regulating outputs with arbitrary relative-degree. This procedure could be applied to design CBFs for constraints with arbitrary relative-degree r. However, this procedure is not directly feasible to $\dot{h}(x,u) = L_f h(x) + L_g h(x) u$ since $L_g h(x)$ is a vector and obviously not invertible (Nguyen and Sreenath (2016)). The work Nguyen and Sreenath (2016) introduces the notion of virtual input-output linearization wherein an invertible decoupling matrix is not required. Considering a virtual control input μ_b defined as

$$h^{(r)}(x,u) = L_f^r h(x) + L_g L_f^{r-1} h(x) u := \mu_b, \tag{29}$$

such that the input-output linearized system becomes (Nguyen and Sreenath (2016))

$$\begin{aligned} \dot{\eta}_b(x) &= F_b \eta_b(x) + G_b \mu_b, \\ h(x) &= C_b \eta_b(x), \end{aligned} \tag{30}$$

where $\eta_b(x)$ is defined as

$$\eta_b(x) := \begin{bmatrix} h(x) \\ \dot{h}(x) \\ \ddot{h}(x) \\ \vdots \\ h^{(r-1)}(x) \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^{r-1} h(x) \end{bmatrix}, \tag{31}$$

$F_b \in \mathbb{R}^{r\times r}$, $G_b \in \mathbb{R}^{r\times 1}$ are defined as

$$F_b = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, G_b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{32}$$

and C_b is defined as

$$C_b = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}. \tag{33}$$

If one wants to drive $h(x)$ to zero, the work Nguyen and Sreenath (2016) proposes to design the EBCF with a pole placement controller $\mu_b = -K_b \eta_b$, with all negative real poles $p_b = -\begin{bmatrix} p_{b_1} & p_{b_2} & \cdots & p_{b_r} \end{bmatrix}$, where $p_{b_i} > 0$, $i = 1, \cdots, r$; thus, $h(x(t)) = C_b e^{A_b t} \eta_b(x_0)$, where the closed-loop matrix $A_b = F_b - G_b K_b$ with all negative real eigenvalues. Moreover, if $\mu_b \geq -K_b \eta_b$, then $h(x(t)) \geq C_b e^{A_b t} \eta_b(x_0)$ (Ames et al. (2019)).

We now can define the ECBF (Ames et al. (2019)), (Nguyen and Sreenath (2016)):

Definition 4. (ECBF) Given a set $\mathcal{C}$ defined by (23) for a r-times continuously differentiable function $h(x) : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}$, then $h(x)$ is an ECBF if there exists a row vector $K_b \in \mathbb{R}^r$ such that for the control system (22),

$$\sup_{u\in\mathcal{U}} \left[L_f^r h(x) + L_g L_f^{r-1} h(x) u \right] \geq -K_b \eta_b(x) \tag{34}$$

$\forall x \in \mathrm{Int}(\mathcal{C})$ results in $h(x(t)) \geq C_b e^{A_b t} \eta_b(x_0) \geq 0$, whenever $h(x_0) \geq 0$.

Similarly to (28), a nominal control law u_{no} and an ECBF $h(x)$ can be unified using QP considering the following controller (Ames et al. (2019)), (Nguyen and Sreenath (2016)):

$$\begin{aligned} \mathbf{u}^*(x) = & \underset{(u,\mu_b)\in\mathbb{R}^{m+1}}{\arg\min} \quad u^T u - 2u_{no}^T u \\ & s.t. \;\; L_f^r h(x) + L_g L_f^{r-1} h(x) u = \mu_b, \\ & \quad\;\; \mu_b \geq -K_b \eta_b(x). \end{aligned} \tag{35}$$

4. NUMERICAL SIMULATIONS

The behavior of the 2DOF helicopter with the control framework described in this work is verified through numerical simulations with MATLAB/Simulink. The numerical values of the parameters are $m_h = 1.317$kg, $l_{cmh} = 0.038$m, $K_{pp} = 0.018$N·m/%, $K_{yy} = -0.0033$N·m/%, $K_{py} = -6.35 \cdot 10^{-4}$N·m/%, $K_{yp} = 10.76 \cdot 10^{-4}$N·m/%, $B_{ph} = 0.1$N/%, $B_{yh} = 0.1$N/%, $J_{eqp} = 0.384$kg·m^2, $J_{eqy} = 0.0432$kg·m^2 (Barbosa et al. (2016)).

As described previously, we apply the proposed control framework to satisfy tracking objectives and safety constraints, where the safety constraints must be prioritized. A LQR is applied as the nominal control law $u_{no_{hel}}$ to ensure that the pitch angle θ_{hel} and the yaw angle ψ_{hel} track the reference inputs θ_{hel_r} and ψ_{hel_r}, respectively, and two safety constraints are considered to ensure that $|\theta_{hel}|$ and $|\psi_{hel}|$ never exceed predetermined bounds θ_{hel_b} and ψ_{hel_b}, respectively.

The LQR is designed for the linearized system (12)-(13). Considering

$$Q = \begin{bmatrix} 550 & 0 & 0 & 0 \\ 0 & 150 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}, \tag{36}$$

$$R = \begin{bmatrix} 5 \cdot 10^{-5} & 0 \\ 0 & 5 \cdot 10^{-5} \end{bmatrix}, \tag{37}$$

and using the MATLAB function *lqr*, we obtain

$$K = \begin{bmatrix} 3305.3 & 142.9 & 1453.7 & 36.2 \\ 273.7 & -1726.1 & 130.6 & -466.6 \end{bmatrix} \tag{38}$$

for the final control law (21).

We apply the safety constraints considering the nonlinear system (3)-(7) described as (22), i.e., $\dot{x}_{hel} = f_{hel}(x_{hel}) + g_{hel}(x_{hel}) u_{hel}$. The safety constraints are given by:

$$h_p = \theta_{hel_b}^2 - \theta_{hel}^2, \tag{39}$$

$$h_y = \psi_{hel_b}^2 - \psi_{hel}^2. \tag{40}$$

These safety constraints have relative-degree two and must be represented as ECBFs. Thus, the QP-based controller (35) that unifies the nominal control law $u_{no_{hel}}$, given by (21), and the safety constraints, expressed as ECBFs,

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is applied. The controller is adapted to multiple safety constraints:

$$\begin{aligned}
\mathbf{u}_{hel}^{*}(x) = & \underset{(u_{hel},\mu_{hel})\in\mathbb{R}^4}{\arg\min} \quad u_{hel}^T u_{hel} - 2u_{no_{hel}}^T u_{hel} \\
s.t.\ & \left[L_{g_{hel}} L_{f_{hel}} h_p(x_{hel}) \ \ L_{g_{hel}} L_{f_{hel}} h_y(x_{hel})\right] u_{hel} \\
& + \left[L_{f_{hel}}^2 h_p(x_{hel}) \ \ L_{f_{hel}}^2 h_y(x_{hel})\right]^T = \mu_{hel}, \\
& \mu_{hel} = \left[\mu_{hel_p} \ \ \mu_{hel_y}\right]^T \geq \\
& -\left[K_{hel_p}\eta_{hel_p}(x_{hel}) \ \ K_{hel_y}\eta_{hel_y}(x_{hel})\right]^T .
\end{aligned} \tag{41}$$

We set $\theta_{hel_b} = \psi_{hel_b} = 0.436\text{rad}\ (25°)$ and $K_{hel_p} = K_{hel_y} = [10\ \ 10]$. The simulation results are presented in Fig. 3, where we consider $\dot{\theta}_{hel_r} = \dot{\psi}_{hel_r} = 0$. From 0s to 50s the reference inputs θ_{hel_r} and ψ_{hel_r} do not exceed the bounds θ_{hel_b} and ψ_{hel_b} and the results show that the LQR is able to track the reference inputs while the CBFs do not exert influence. After 50s the reference inputs θ_{hel_r} and ψ_{hel_r} exceed the bounds θ_{hel_b} and ψ_{hel_b} and the safety constraints are respected, i.e, $|\theta_{hel}|$ never exceeds θ_{hel_b}, $|\psi_{hel}|$ never exceeds ψ_{hel_b}, and the safe set (23) is respected ($h_p \geq 0$ and $h_y \geq 0$). From 50s and 80s there is a conflict between the safety constraints and the reference inputs, i.e, they cannot be satisfied simultaneously. As described previously, this control framework prioritizes safety constraints, therefore the safety constraints are respected and the reference inputs are not tracked adequately. It is important to highlight that all the simulations were done considering the nonlinear system (3)-(7) and the QP is implemented using Hildreth's QP procedure (Hildreth (1957)).

5. CONCLUSIONS

This work presents the safe control of a 2DOF helicopter considering a control framework that unifies stability/tracking objectives, expressed as a nominal control law, and high relative-degree safety constraints, expressed as ECBFs, through QP and the safety constraints must be prioritized. A LQR is applied as the nominal control law and two safety constraints are considered to ensure that pitch and yaw angles never exceed predetermined bounds. The system model is based on a 2DOF helicopter available at EPUSP. The numerical simulations demonstrate that the safety constraints are always satisfied and the tracking objectives are satisfied just when are not in conflict with the safety constraints. As a future work, the control framework will be implemented experimentally in the 2DOF helicopter available at EPUSP.

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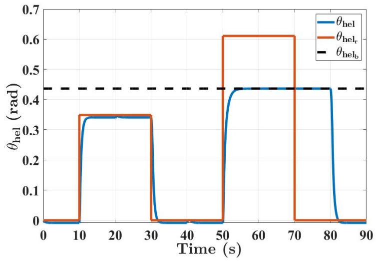
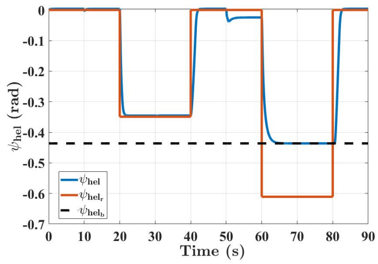
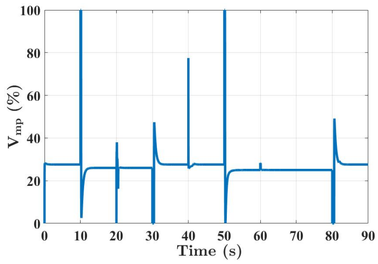


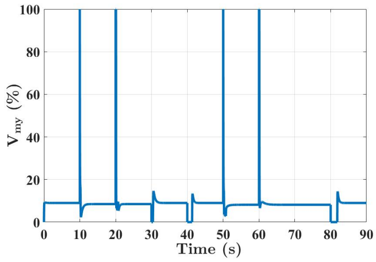
(a) θ_{hel}



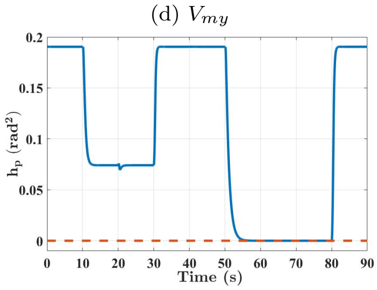
(b) ψ_{hel}



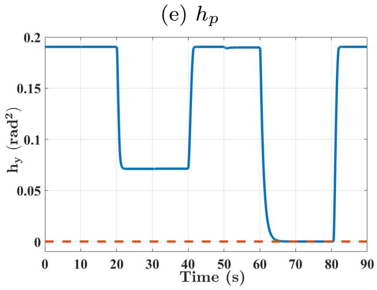
(c) V_{mp}



(d) V_{my}



(e) h_p



(f) h_y

Fig. 3. Simulation results - LQR with ECBFs.

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