Pulse Pattern Optimization for Selective Harmonic Elimination in Multilevel Inverters using Genetic Algorithms ⋆

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Abstract: This paper presents the application of genetic algorithm to determine optimized pulse patterns in multilevel inverters aiming to mitigate harmonics generated at the inverter output voltage while maintaining a desired modulation index. A new real representation based on a relative position approach is applied to the chromosome encoding in order to avoid the existence of infeasible solutions and a systematic diversity control of initial population is proposed based on the euclidean distance between individuals. A local search mechanism combined with an elitist selection is also considered. Fitness value frequency distribution, waveform patterns and its corresponding harmonic components are presented to illustrate the effectiveness of the proposed algorithm.

Keywords: Genetic algorithms, Multilevel inverters, Selective harmonic elimination, Optimization

1. INTRODUCTION

In the last years, multilevel inverters have drawn attention as a solution for power conversion in high voltage applications. This inverter topology allows, by adding different DC levels, the generation of high output voltages with stepped waveforms while applying reduced voltages to its switches (Rodríguez et al., 2002). In general, inverter efficiency is reduced by switching losses, giving rise to low-frequency driving strategies (Dahidah and Agelidis, 2008; Qian et al., 2018). One of the main drawbacks of these strategies are the high harmonic content generated at the inverter output voltage. These harmonics could be filtered, but that would increase the inverter complexity and fabrication cost. An alternative to reduce output voltage harmonics lies in the correct choice of the pulse pattern (switching angles) applied to the inverter switches (Rathore et al., 2010). To accomplish that, the PWM Selective Harmonic Elimination (SHE) modulation strategy can be used.

The main challenge in the SHE technique is to choose the right switching angles to provide correct harmonic elimination while achieving the desired amplitude at the fundamental harmonic component. In order to solve this problem different approaches based on heuristic methods are reported in the literature. In Dahidah and Agelidis (2008) a generalized formulation is developed and a Genetic Algorithm (GA) is applied on the pulse pattern optimization of five and seven-level inverters. The use of GA is also reported in Zhang et al. (2014), Dahidah and Rao (2007), Omara et al. (2018), Chatterjee et al. (2017), Kumar et al. (2017), de Almeida et al. (2016) and Bindu et al. (2011).

Besides the genetic algorithm, other heuristic methods have been applied to the SHE switching pattern optimization. Ali et al. (2017) applies a Particle Swarm Optimization (PSO) algorithm to obtain the switching angles of an eleven level inverter. Memon et al. (2018) used a variation of the same algorithm, where a Newton-Raphson method was combined with PSO to include a local search and refine the results. To solve the same problem the Bee Algorithm have been explored by Kavousi et al. (2012) and the Memetic Algorithm was applied by Niknam Kumle et al. (2015). More recently, a comparative study between different optimization techniques to find precise switching angles for SHE modulation was made by Kundu et al. (2018). The GA was used by Iqbal et al. (2019) to solve the SHE optimization problem for a seven-level packed U-cell inverter and Kumar et al. (2020) applied a Gravitational Search Algorithm and a Learning PSO to obtain optimized switching angles for seven and eleven level inverters.

In this paper, the GA is applied to the optimization problem of selecting switching angles for selective harmonic elimination. A relative position approach is proposed to represent the switching angles in the GA, eliminating the occurrence of infeasible solutions. Also, the proposed representation allows a systematic diversity control of the initial population which guarantees that it is spread in the solution space, increasing the chances of

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reaching global rather than local minimum. In addition, to avoid the algorithm to converge into nearly optimum solutions, a local search mechanism is implemented. To validate the effectiveness of the proposed algorithm, the optimization problem was solved one thousand times for unit modulation index and the frequency distribution of solutions was analyzed. Furthermore, the output voltage waveform, output's harmonic content and evolution of the fitness throughout the generations are presented for the best solution achieved. The frequency distributions of the algorithm's solution for different modulation indexes are also presented.

2. MULTILEVEL INVERTER

One possible topology for multilevel inverters, as presented in Rodríguez et al. (2002), is the series connection of single-phase inverters. In this work, to obtain a seven-level inverter, the same topology considered by Dahidah and Agelidis (2008) is used: three cascaded full bridge inverters with separated DC sources, as shown in Figure 1. Without loss of generality, all DC sources are assumed to be the identical.

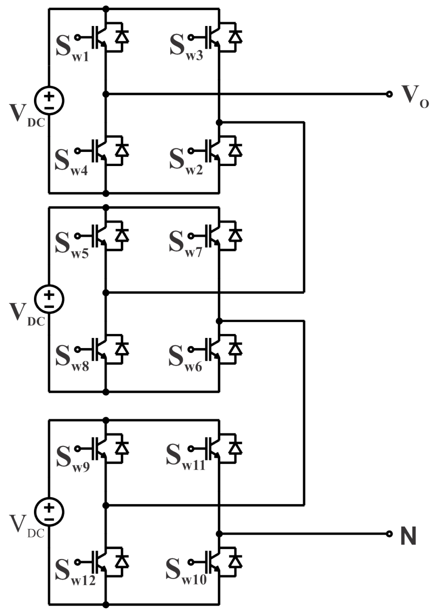

Figure 1. Seven level cascaded inverter

As pointed by Rodríguez et al. (2002), each single-phase full bridge generates one of three possible outputs voltages, $+V_{DC}$, 0 and $-V_{DC}$. By correct switching of S_{w1} to S_{w12}, the voltage of each full-bridge can be added to the output, generating seven different levels and a nearly sinusoidal staircase waveform. A possible combination of switch states and the resulting output voltage levels are shown in Table 1. In addition, a three-phase output voltage can be obtained by the individual control of each inverter with switching sequences separated by 120°.

Even though multilevel inverters are able to synthesize nearly sinusoidal output voltages, the converter still suffers from harmonic problems that could be a limitation for several applications. As stated by Chatterjee et al. (2017) triplen harmonics (3^{rd}, 9^{th} and 15^{th}) oscillate the current in the neutral wire increasing its current, while 5^{th} harmonics produce a negative torque when connected to a motor.

Table 1. Switches states for each voltage level.

V_o	S_{w1}	S_{w2}	S_{w3}	S_{w4}	S_{w5}	S_{w6}	S_{w7}	S_{w8}	S_{w9}	S_{w10}	S_{w11}	S_{w12}
$3V_{DC}$	1	1	0	0	1	1	0	0	1	1	0	0
$2V_{DC}$	1	1	0	0	1	1	0	0	0	1	0	1
V_{DC}	1	1	0	0	0	1	0	1	0	1	0	1
0	0	1	0	1	0	1	0	1	0	1	0	1
$-V_{DC}$	0	0	1	1	1	0	1	0	1	0	1	0
$-2V_{DC}$	0	0	1	1	0	0	1	1	1	0	1	0
$-3V_{DC}$	0	0	1	1	0	0	1	1	0	0	1	1

As presented in Kumar et al. (2017), n switching angles in a quarter of cycle allow to eliminate $n-1$ low-order harmonics while controlling the fundamental component. Due to the quarter-wave symmetry of the output voltage waveform, even harmonics are null. Hence, to mitigate up to the 17^{th} harmonic (eight odd harmonic components) as done by Dahidah and Agelidis (2008), nine switching events per quarter cycle of the wave ($n = 9$) were used. Figure 2 illustrates a synthesized waveform from an one-phase seven level inverter using equally spaced switching angles and its corresponding harmonic spectrum in which the presence of harmonic components can be noticed.

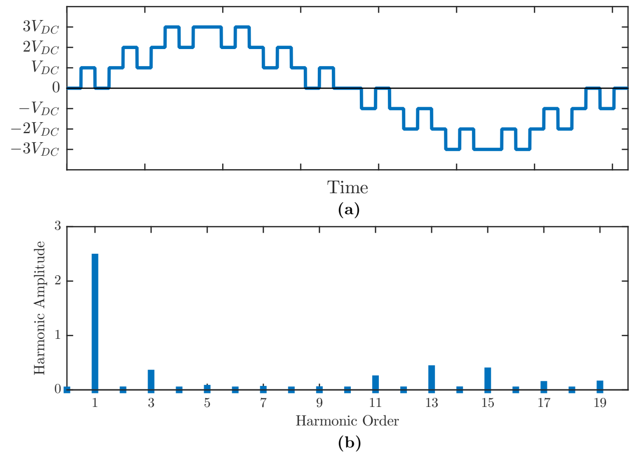

Figure 2. (a) Seven level cascaded inverter output waveform. (b) Output voltage spectrum.

Assuming the output waveform is quarter-wave symmetrical, the SHE challenge is to decide switching angles $\alpha_i,\ i = 1, ..., n$, satisfying

$$0 \leq \alpha_1 \leq \alpha_2 \leq ... \leq \alpha_n \leq \frac{\pi}{2} \quad (1)$$

that mitigates the desired harmonics while achieving a required fundamental component amplitude, as illustrated in Figure 3 ($n = 9$). The switching angles for desired operation points can be computed through offline optimization methods and then used to control the inverter.

3. GENETIC ALGORITHM

The following section is dedicated to describe the proposed genetic algorithm implementation.

3.1 *Chromosome Encoding*

Chromosome encoding is how switching angles α_i are represented. Binary encoding was used by Zhang et al. (2014) (17 bits per angle) and de Almeida et al. (2016)

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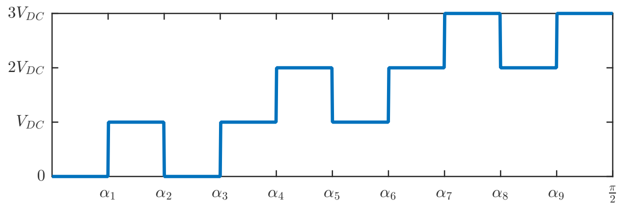

Figure 3. Seven level cascaded inverter output quarter-wave

(21 bits per angle). Real (floating point) encoding was used by Dahidah and Rao (2007) and Omara et al. (2018). As pointed by Michaelwicz (1996), real encoding is generally better in precision when representing large domains and handling constraints.

Every representation proposed by these works explicitly considered inequality (1) for n switching angles. During the execution of the GA, new individuals that do not meet the constraint (1) can be generated. These individuals must be discarded, wasting in this way computation time, or rearranged, allowing different representations for the same solution.

This work proposes a novel real encoding where infeasible solutions cannot be represented and with an unique representation of a given solution. Each chromosome is encoded by a vector $S \in \mathbb{R}^n$, with $0 < S_i < 1$ for $i = 1, ..., n$. The i-th switching angle α_i is calculated from the chromosome S by

$$\alpha_i = \begin{cases} S_i \dfrac{\pi}{2} \mid i = 1 \\ \\ S_i \left(\dfrac{\pi}{2} - \alpha_{i-1}\right) + \alpha_{i-1} \mid i \neq 1, \end{cases} \tag{2}$$

where S_i is the i-th gene of a chromosome. So, S_i represents the position of α_i with respect to the interval between the previous angle (α_{i-1}) and $\pi/2$ in such way that every switching angle α_i is guaranteed to be greater than α_{i-1} and less than $\pi/2$, thus verifying condition (1).

To better illustrate, a small example is given: consider that $S = \{0.2, 0.6, 0.4\}$ is a given individual from a $n = 3$ switching angles problem. Converting this individual's chromosome through (2) results,

$$\alpha_1 = 0.2\frac{\pi}{2} = 0.31 \text{ rad}, \tag{3}$$

$$\alpha_2 = 0.6\left(\frac{\pi}{2} - 0.31\right) + 0.31 = 1.07 \text{ rad}, \tag{4}$$

$$\alpha_3 = 0.4\left(\frac{\pi}{2} - 1.07\right) + 1.07 = 1.27 \text{ rad}, \tag{5}$$

in which is noticed that the switching angles obtained satisfies (1).

3.2 Fitness Function

The objective of SHE optimization problem is to decide the n switching angles α_i which minimize a criteria $f(S)$ called fitness function. In this work, $f(S)$ is given by

$$f(S) = (F_1 - A_0)^2 + F_3^2 + F_5^2 + F_7^2 + ... + F_{17}^2 \tag{6}$$

where F_j is the j-th normalized harmonic component obtained from the Fourier transform of a waveform constructed from a solution S and A_0 is the desired normalized fundamental component. For N DC sources, the term A_0 is computed by

$$A_0 = \frac{N\pi m}{4}, \tag{7}$$

where the modulation index $m \in \mathbb{R}$, $0 \leq m \leq 1$, represents the desired ratio between the output waveform fundamental amplitude and the maximum fundamental amplitude achievable. For high-power voltage source multilevel inverters, the range of m usually lies between 0.7 to 1.0 (Dahidah and Rao, 2007).

Weighting factors can be added to each component of (6) to increase (or decrease) the relevance of a specific harmonic in the proposed fitness function. However, in this work, all harmonic components were equally weighted.

3.3 Initial Population

In most works, e.g., Dahidah and Agelidis (2008), Bindu et al. (2011), Zhang et al. (2014) and Chatterjee et al. (2017), the solution space is first randomly populated. This does not prevent the coexistence of similar individuals in the initial population. To avoid this problem, an heuristic is proposed to guarantee diversity control of initial population by using euclidean distance. Given two solutions S^a and S^b, the distance d is computed by

$$d = \sqrt{\sum_{i=1}^{n} (S_i^a - S_i^b)^2}, \tag{8}$$

with S_i^a and S_i^b being the i-th element of each solution. An initial population is considered valid if the following criteria are met:

$$\begin{cases} d > d_t \ \forall S^a, S^b \mid S^a \neq S^b \\ \min(S_i^a) > d_t/2 \ \forall S^a \\ \max(S_i^a) < 1 - d_t/2 \ \forall S^a. \end{cases} \tag{9}$$

In this case, d_t is the minimum distance allowed between any two solutions. Each solution defines then a hypersphere of dimension n and diameter d_t, such that (9) is equivalent to:

- hyperspheres defined by any two given solutions cannot intersect each other;
- hyperspheres defined by any given solution cannot intersect the boundary of the solution space.

The choice of d_t is arbitrary, however upper bounded by the volume of the solution space, which is an unit hypercube for the proposed representation. From works about hypersphere packing (Conway and Sloane, 1999), considering N_{pop} individuals within the initial population, the maximum packing density η_{max}, is:

$$\eta_{max}(N_{pop}, d_t) = N_{pop}\left(\frac{2^n \pi^{\left(\frac{n-1}{2}\right)} \left(\frac{n-1}{2}\right)!}{n!}\right)\left(\frac{d_t}{2}\right)^n. \tag{10}$$

It is important to notice that for the problem addressed in this work, $n = 9$, the maximum packing density is $\eta_{max} = 0.1457$ (Conway and Sloane, 1999), resulting in a maximum d_t given by:

$$d_t \leq d_{tmax} = 1.4142\sqrt[9]{\frac{1}{N_{pop}}}. \tag{11}$$

It is also important to notice that the maximum density packing is only achieved by means of optimal packing, since for d_t approaching d_{tmax} it is increasingly difficult for a random algorithm to populate the solution space and then a smaller distance should be adopted.

3.4 Mutation

Mutation is the operation by which an individual is randomly changed. In the proposed chromosome encoding every gene has a different influence in the represented solution. For example, changing the first gene alters all switching angles. To avoid a possible negative impact in the mutation operator, a conversion between representations is made before mutation. The following example illustrates the proposed operator:

(1) Assume that an individual is selected for mutation, e.g. $S = \{0.30, 0.10, 0.40, 0.10, ...\}$;
(2) The selected individual's chromosome is converted using relation (2) to switching angles, resulting in α = {0.47, 0.58, 0.98, 1.04, ...} radians;
(3) One random switching angle is changed to a random value uniformly distributed in the interval $(0, \pi/2)$. In this case, it is considered that α_1 is changed from 0.47 to 0.64 rad, resulting in $\alpha = \{0.64, 0.58, 0.98, 1.04, ...\}$ rads;
(4) The switching angles are sorted in ascending order, resulting in $\alpha = \{0.58, 0.64, 0.98, 1.04, ...\}$ rad;
(5) Switching angles are represented back to the chromosome encoding ($S = \{0.37, 0.06, 0.37, 0.10, ...\}$). The resultant mutated solution S had only one switching angle altered, although multiple genes have been changed.

The number of individuals created by mutation, N_m, is a function of the generation n_{ger} and given by:

$$N_m = R_{int}\left\{N_{m0} + \frac{n_{ger}}{N_{ger}}(N_{mf} - N_{m0})\right\}. \tag{12}$$

In (12), $R_{int}\{.\}$ represents a function that rounds the argument to its nearest integer, N_{ger} is the maximum number of generations, N_{m0} is the initial number of mutations and N_{mf} if the final number of mutations. To select the individual for mutation, a group of N_{mt} is randomly chosen for tournament and the best solution from this group is used in the mutation algorithm.

3.5 Crossover

Crossover is the operation by which an offspring is generated using the information of parents. In this case, uniform crossover is considered as follows: given two different parents S^a and S^b, two descendants O^a and O^b are generated as

$$\begin{cases} \text{if } \chi_i > 0.5 \text{ then } O_i^a = S_i^a \text{ and } O_i^b = S_i^b \\ \text{if } \chi_i \leq 0.5 \text{ then } O_i^a = S_i^b \text{ and } O_i^b = S_i^a, \\ \forall i = 1, ..., n, \end{cases} \tag{13}$$

where χ_i is a real random number uniformly distributed between 0 and 1 and S_i^a, S_i^b, O_i^a and O_i^b are the i-th gene of the parents and descendants, respectively. The complete crossover operation, which happens at the end of each generation, is described as follows:

(1) Two different groups of N_{ct} individuals each are randomly chosen in the population for tournament;
(2) The best solution of each group is used as a parent;
(3) From the parents two individuals are generated using (13).

The number of individuals created by crossover, N_c, changes along generations following,

$$N_c = R_{even}\left\{N_{c0} + \frac{n_{ger}}{N_{ger}}(N_{cf} - N_{c0})\right\}, \tag{14}$$

where $R_{even}\{.\}$ represents a function that rounds the argument to its nearest even integer, N_{c0} is the initial number of individuals generated by crossover and N_{cf} is the final number individuals generated by crossover.

3.6 Elitism and Local Search

Elitism is the mechanism by which the fittest individuals of the previous generation are copied to the next. The work of Dahidah and Agelidis (2008) used direct local search in the neighborhood of the fittest solution in an attempt to improve the result. The considered neighborhood was randomly generated and decreases along with the steps.

This work uses elitism combined with direct local search, proposing a different method than the one in (Dahidah and Agelidis, 2008). A local search is performed by a Hill Climbing Algorithm (HCA) described as follows:

(1) At the end of a generation, identify the best solution (minimal fitness value);
(2) Convert this solution to switching angles α_i;
(3) Generate $2n$ candidate solutions in the neighborhood of α_i by adding and subtracting $\Delta\alpha_i$ to each gene, where n is the number of switching angles;
(4) Evaluate the fitness of the $2n + 1$ available solutions (the best and its $2n$ neighbors);
(5) If a neighbor is better than the previous best solution by a margin larger than ϕ percent, this neighbor is considered a new best solution and the algorithm returns to step 3. Otherwise, the local search is terminated and the best switching angles are converted to the chromosome encoding.

The proposed HCA is deterministic and the distance from the current best solution to the evaluated neighbors can be controlled by an adequate choice of $\Delta\alpha_i$.

4. RESULTS

Table 2 shows defined parameters for the developed genetic algorithm to obtain optimized switching angles for the seven-level cascaded inverter showed in Figure 1. Numerical values of the parameters were chosen based on preliminary experiments. Without loss of generality, all DC sources were considered equal and normalized to $V_{DC} = 1$ pu. The presented algorithm can be extended to an unequal sources scenario simply by considering the different voltage levels on the evaluation of the fitness function.

4.1 Selective Harmonic Elimination Analysis

Based on the parameters described in Table 2, the proposed GA was solved and the resulting switching angles

Table 2. Parameters for Genetic Algorithm.

Parameter	Symbol	Value
Max. number of generations	N_{ger}	70
Population size	N_{pop}	200
Crossover tournament size	N_{ct}	3
Initial crossover generated indiv.	N_{c0}	120
Final crossover generated indiv.	N_{cf}	20
Mutation tournament size	N_{mt}	2
Initial mutation generated indiv.	N_{m0}	20
Final mutation generated indiv.	N_{mf}	120
Distance between initial indiv.	d_t	0.40
Angle for HCA neighborhood	$\Delta\alpha_i$	0.0035 rad
Modulation index	m	1
Desired fundamental amplitude	A_0	2.36 p.u.
Tolerance margin for local search	ϕ	1%

are presented in Table 3. For this scenario, the SHE-PWM pattern, normalized by V_{DC}, and its corresponding harmonic components are shown in Figure 4.

Table 3. Optimal switching angles for $m = 1$.

Angle	Value (rad)	Angle	(rad)
α_1	0.0142	α_6	0.8248
α_2	0.1034	α_7	1.2673
α_3	0.2527	α_8	1.3368
α_4	0.6257	α_9	1.5238
α_5	0.7419		

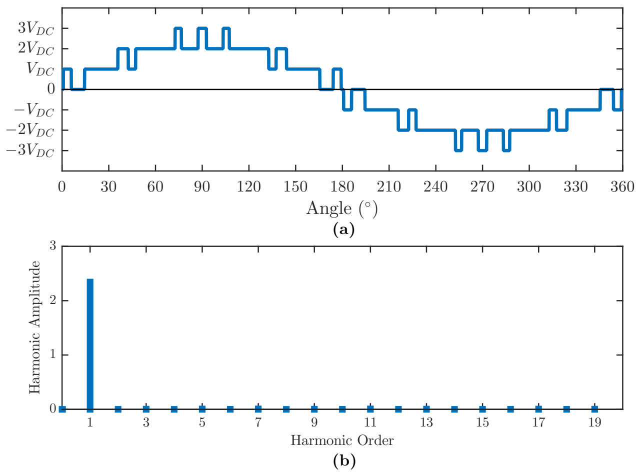

Figure 4. (a) Optimized output voltage (p.u) for Unit Modulation Index, (b) Output voltage spectrum.

It can be noticed in Figure 4 that the required fundamental amplitude was achieved (the corresponding fundamental amplitude is $A_0 = 2.36\ pu$ for $m = 1$) while maintaining the undesired harmonics in a negligible range (highest harmonic component with amplitude 0.0103 pu). This results illustrates the effectiveness of the developed algorithm in terms of harmonic mitigation.

4.2 Genetic Algorithm Convergence Analysis

To analyze the convergence of the developed GA, the algorithm was first executed one thousand times for a unit modulation index and its final fitness values were observed. Histogram in Figure 5 shows the relative distribution of results. The algorithm was able to achieve fitness in the first bin of the histogram (below 0.0036) 45% of the time, with the lowest fitness being 4.7x10^{-4}. The algorithm was executed in MATLAB on Ubuntu with Intel Core i3-6006U and 4GB memory and its average execution time was 5.37 seconds.

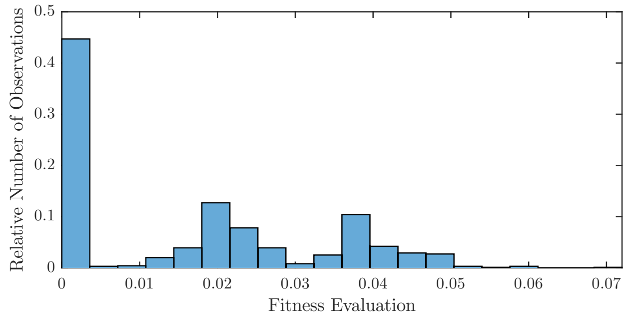

Figure 5. Distribution of fitness function evaluation achieved in 1000 executions for $m = 1$

The fitness evolution throughout the generations for the solution with lowest fitness is given in Figure 6. Note that while crossover is responsible for a fast improvement of the best individual within few generations, mutation is necessary for late changes in individuals. Those late changes combined with a local search mechanism are necessary to escape a local minimum.

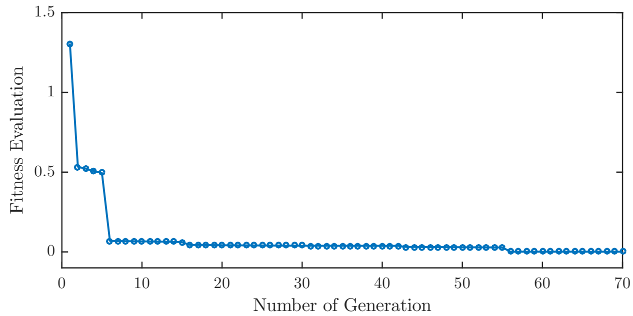

Figure 6. Fitness evolution throughout the generations

The proposed algorithm was also executed one hundred times for modulation indexes from 0.70 to 1.00 with steps of 0.05. The distribution of results is represented in the boxplot of Figure 7, where:

- red line represents the sample median;
- the bottom of the box represents the first quartile;
- the top of the box represents the third quartile;
- lower whisker is the lowest value inside 1.5 times the interquartile range;
- upper whisker is the highest value inside 1.5 times the interquartile range;
- red crosses represent outliers (data out of the interquartile range).

In Figure 7 it is observed that the median are generally closer to the bottom of interquartile range and there are few outliers below the distribution. This indicates that the algorithm is capable of achieving its best results consistently.

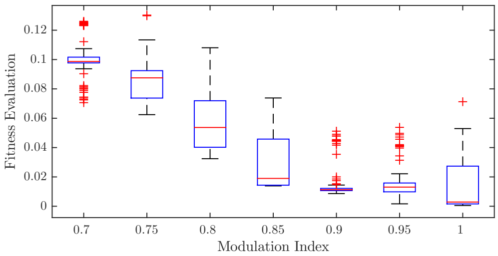

Figure 7. Distribution of fitness evaluation achieved in 100 executions for different modulation indexes

5. CONCLUSION

In this paper, an application of GA for switching angles optimization in selective harmonic elimination was developed. A real chromosome encoding was proposed, with the advantage of representing only feasible solutions and allowing a systematic diversity control of initial population. The genetic operators of mutation and crossover were also described in detail.

Switching angles patterns for a seven-level cascaded full-bridge inverter with equal DC sources were obtained and its patterns were presented to validate the algorithm. Statistical analysis was provided, indicating that the algorithm achieves its best results within a few executions. This feature indicates the proposed GA can be used during the operation of the inverters, where new optimizations may be performed due to changes in conditions.

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