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Robust Static Output Feedback Stabilization of Linear Systems Using Dissipativity Theory

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Abstract: This paper deals with robust static output feedback (SOF) stabilization of linear time-invariant (LTI) systems with transient performance. The proposed approach considers uncertainties on the system matrices and does not impose any constraints on the output matrix. We use the definition of strict QSR-dissipativity to formulate new sufficient conditions in the form of linear matrix inequalities (LMIs) for asymptotic stabilization. One of the main advantages of the developed strategy is that in many cases static output feedback can be designed in a non-iterative manner. Numerical examples highlight the effectiveness of the proposed approach.

Keywords: Static output feedback, Robust control, Linear systems, Dissipativity.

1. INTRODUCTION

A linear static output feedback (SOF) gain is a very simple controller in comparison with other control methods. It can be applied when it is not possible to access all the plant state information (Sadabadi and Peaucelle, 2016). Designing a SOF is a challenging problem in control theory and, in general, it results in non-convex conditions which cannot be solved through linear matrix inequalities (LMIs). Over the last decades, many papers have proposed iterative and non-iterative strategies to deal with this problem through LMI conditions. See Veselý (2001), Crusius and Trofino (1999), Apkarian and Noll (2006), Gahinet and Apkarian (2011), Sadabadi and Peaucelle (2016) for an overview of the subject.

In this work, we use the concept of dissipativity to formulate a strategy for the linear SOF design. Dissipativity theory was introduced some decades ago and provides some mathematical definitions for general input-affine systems (Willems, 1972; Brogliato et al., 2020). Dissipative systems can also be Lyapunov stable, asymptotic or even exponentially stable. For these reasons, dissipativity theory has been extensively used in stability analysis and control systems design (Brogliato et al., 2020). Dissipativity is a generalization of the notion of passivity and numerous works in the literature have proposed passivity- and dissipativity-based stabilization strategies (Hill and Moylan, 1976; Astolfi et al., 2002; Feng et al., 2013; Ortega and Garcia-Canseco, 2004; Shishkin and Hill, 1995). Madeira and Viana (2020) applied the definition of strict QSR-dissipativity to formulate sufficient LMI conditions for the design of a robust SOF gain that stabilizes rational or polynomial nonlinear systems. In Madeira (2021), it was proved that strict QSR-dissipativity, under mild assumptions, is a necessary and sufficient condition for SOF stabilizabity of LTI systems.

This paper uses the same definition of dissipativity considered in Madeira and Viana (2020). We propose a framework for the robust SOF design for stabilization of uncertain linear systems. The proposed strategy provides new sufficient LMI conditions for feedback stabilization with a lower bound on the decay rate that guarantees transient performance, as in Sereni et al. (2018). Differently from most papers dealing with SOF design, our strategy provides LMI conditions to deal with this problem, which in many cases can be solved in a non-iterative manner. In addition, we do not impose any restriction on the output plant matrix.

This paper is organized as follows. In section 2, we present the problem formulation. In section 3, some preliminaries used to obtain the conditions are presented. In section 4, the main result of the paper is shown. In section 5, some numerical examples are provided to illustrate the effectiveness of the strategy. Finally, in section 6, we have the conclusion of the paper.

Notation. For a matrix $H \in \mathbb{R}^{n\times m}$, $H^\top \in \mathbb{R}^{m\times n}$ means its transpose. Operators $H \succ 0$ and $H \succeq 0$ means that the symmetric matrix H is positive definite or positive semidefinite, respectively. $He\{A\}$ stands for $A + A^\top$. I_m is the $m\times m$ identity matrix. For a polytope $\mathcal{B}$, $\mathcal{V}(\mathcal{B})$ is the set of vertices of $\mathcal{B}$ and $\mathcal{V}(\mathcal{B})_i$ is the i_{th} vertex of the polytope. For a symmetric block matrix, the symbol $*$ stands for the transpose of the blocks outside the main diagonal block.

2. PROBLEM FORMULATION

2.1 Uncertain LTI system

Consider an uncertain LTI system as presented in Bernussou et al. (1989),

$$\begin{cases} \dot{x}(t) = A(\delta)x(t) + B(\delta)u(t), \\ y(t) = C(\delta)x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the measured output. Moreover, $\delta \in \mathcal{D} \subseteq \mathbb{R}^q$ is a vector of system uncertainties and $A(\delta) \in$

$\mathbb{R}^{n\times n}, B(\delta) \in \mathbb{R}^{n\times m}, C(\delta) \in \mathbb{R}^{p\times n}$ are uncertain matrices affine on δ. The uncertainties are bounded, then the vector δ lies inside a polytope $\mathcal{D}$ of $N = 2^q$ vertices, where q is the number of elements of δ. In addition, δ can be related with a set of constants α_i by $\delta = \sum_{i=1}^{N} \alpha_i \mathcal{V}(\delta)_i$. Then, the polytope $\mathcal{D}$ can be defined in terms of $\alpha = \{\alpha_1, \dots, \alpha_N\}$ as an unitary simplex (Oliveira and Peres, 2007),

$$\Omega = \{\alpha(\delta) \in \mathbb{R}^N : \sum_{i=1}^{N} \alpha_i = 1; \alpha_i \geq 0; i = 1, \dots, N\}. \quad (2)$$

Moreover, matrices $A(\delta), B(\delta)$ and $C(\delta)$ can be represented in a polytopic domain

$$\Theta = \{(A, B, C)(\delta) = \sum_{i=1}^{N} \alpha_i (A, B, C)_i,\ \alpha \in \Omega\}. \quad (3)$$

2.2 Problem Statement

The problem we intend to solve can be summarized as follows.

Problem 1. Find a static output feedback gain K, i.e., a control law $u(t) = Ky(t)$, such that the closed-loop system given by

$$\dot{x}(t) = (A(\delta) + B(\delta)KC(\delta))x(t) \quad (4)$$

is asymptotically stable for all $\delta \in \mathcal{D}$ with a lower bound on the decay rate given by γ.

3. PRELIMINARIES

Here, we present some definitions that are necessary to formulate the strategy. The following lemma will be used on the demonstration of our main results in section 4.

Lemma 2. If the following LMIs hold

$$Y_{ii} \prec 0, \text{ for } i = 1, 2, ..., N, \quad (5)$$

$$Y_{ij} + Y_{ji} \prec 0, \text{ for } 1 \leq i < j \leq N, \quad (6)$$

then it is true that

$$\sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} \alpha_j Y_{ij} \prec 0. \quad (7)$$

Proof. See Tanaka et al. (1998).

3.1 Decay rate

The decay rate is a performance index associated with the system transient duration. As presented in (Boyd and Vandenberghe, 2004), the mathematical definition of decay rate is the largest γ such that

$$\lim_{t\to\infty} e^{\gamma t} ||x(t)|| = 0, \quad (8)$$

holds for all trajectories of the vector $x(t)$.

A lower bound on the decay rate γ can be computed at the same time that system stability is ensured if we consider a quadratic Lyapunov function ($V(x) = x^\top P x \succ 0$), such that

$$\dot{V}(x) \leq -2\gamma V(x), \quad (9)$$

holds for all trajectories of $x(t)$, with $\gamma > 0$ (Boyd and Vandenberghe, 2004).

3.2 Dissipativity

Consider, first, the following LTI system without uncertainties

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t). \end{cases} \quad (10)$$

System (10) is said to be dissipative if it is completely reachable and there exists a nonnegative storage function $V(x(t))$, where $V : \mathcal{X} \to \mathbb{R}$ and $V \in C^1$, and a locally integrable supply rate $r(u(t), y(t))$ such that $\dot{V} \leq r(u, y)$ (Haddad and Chellaboina, 2011). Further definitions of dissipativity can be found in Brogliato et al. (2020). In this work, we mainly apply the definition of strict QSR-dissipativity, which is defined below.

Definition 3. If there exists a storage function $V(x) \succeq 0$, a supply rate $r(u, y) = y^\top Q y + 2y^\top S u + u^\top R u$, and a function $T(x) \succ 0$ such that

$$\dot{V}(x) + T(x) \leq y^\top Q y + 2y^\top S u + u^\top R u, \quad (11)$$

then system (10) along all possible trajectories starting at $x(0)$, for all $t \geq 0$, is said to be *strictly QSR-dissipative*. $S \in \mathbb{R}^{p\times m}$ is real matrix and $Q \in \mathbb{R}^{p\times p}, R \in \mathbb{R}^{m\times m}$ are real and symmetric matrices.

In practice, condition (11) express that only a fraction of the energy supplied from $r(u, y)$ is stored by the dissipative system. Also, only a fraction of its stored energy $V(x)$ can be delivered to its surroundings. Moreover, Definition 3 can be related with system stability. If a system is strictly QSR-dissipative with $V(x) \succ 0$ and $Q \preceq 0$, then the free system is asymptotically stable (Haddad and Chellaboina, 2011). In this work, we consider quadratic Lyapunov functions $V(x)$

$$V(x) = x^\top P x, \quad P \succ 0, \quad (12)$$

and, to compute a lower bound on the decay rate of the system, we restrict $T(x)$ to be a multiple of $V(x)$

$$T(x) = 2\gamma V(x), \quad \gamma > 0. \quad (13)$$

In the following, the application of dissipativity for static output feedback stabilization of linear systems is presented by some lemmas.

Lemma 4. (Haddad and Chellaboina (2011)) The LTI system (10) is said to be strictly QSR-dissipative if the following LMI holds

$$\begin{bmatrix} A^\top P + PA + H - C^\top QC & PB - C^\top S \\ B^\top P - S^\top C & -R \end{bmatrix} \preceq 0, \quad (14)$$

for some symmetric matrices $Q, P \succ 0, R \succ 0$, and matrices $S, H \succ 0$.

Lemma 5. (Madeira (2021)) The LTI system (10) is linear SOF stabilizable if and only if (14) is satisfied with symmetric matrices $Q, P \succ 0, R \succ 0$, matrices $S, H \succ 0$, and $\Delta = 0$, where

$$\Delta = SR^{-1}S^\top - Q, \quad (15)$$

and a stabilizing gain is given by

$$K = -R^{-1}S^\top. \quad (16)$$

For the uncertain LTI system (1), dissipativity condition (11) can be rewritten as

$$\begin{aligned} t(x, u, \delta) = \nabla V^\top [A(\delta)x + B(\delta)u] + 2\gamma V \\ -y^\top Q y - 2y^\top S u - u^\top R u \leq 0, \end{aligned} \quad (17)$$

and system (1) is said to be *robust strictly QSR-dissipative* if $t(x, u, \delta) \leq 0$ for all $\delta \in \mathcal{D}$.

4. STATIC OUTPUT FEEDBACK DESIGN

In this section, Theorem 6 presents the proposed strategy that uses strictly QSR-dissipativity and Lemma 2 to solve Problem 1.

Theorem 6. Let $\mathcal{D}$ be a polytope of δ, described by (2). Given some $\gamma > 0$, suppose that there exists symmetric matrices $P \succ 0 \in \mathbb{R}^{n\times n}$, $R \succ 0 \in \mathbb{R}^{m\times m}$, $Q \in \mathbb{R}^{p\times p}$ and a matrix $S \in \mathbb{R}^{p\times m}$ such that

$$Y_{ii} \prec 0, \text{ for } i = 1, \ldots, N, \tag{18}$$

$$Y_{ij} + Y_{ji} \prec 0, \text{ for } 1 \leq i < j \leq N, \tag{19}$$

and

$$\Delta = SR^{-1}S^\top - Q \succeq 0, \tag{20}$$

where Y_{ij} is given by

$$\begin{bmatrix} PA_i + A_i^\top P + 2\gamma P - C_i^\top QC_j & * \\ B_i^\top P - S^\top C & -R \end{bmatrix} \tag{21}$$

then,

(i) System (1) is strictly QSR-dissipative for all $\delta \in \mathcal{D}$.
(ii) The SOF given by
$$K = -R^{-1}S^\top, \tag{22}$$
asymptotically stabilizes (1) for all $\delta \in \mathcal{D}$ around the origin with a lower bound on the decay rate given by γ.

Proof. First, if conditions (18) and (19) are satisfied, then by Lemma 2, the following holds

$$\sum_{i=1}^{N}\alpha_i\sum_{j=1}^{N}\alpha_j Y_{ij} = \begin{bmatrix} \sum_{i=1}^{N}\alpha_i\sum_{j=1}^{N}\alpha_j\Pi_{ij} & * \\ \sum_{i=1}^{N}\alpha_i\sum_{j=1}^{N}\alpha_j\Pi_i & \sum_{i=1}^{N}\alpha_i\sum_{j=1}^{N}\alpha_j(-R) \end{bmatrix} \prec 0, \tag{23}$$

where $\Pi_{ij} = PA_i + A_i^\top P + 2\gamma P - C_i^\top QC_j$ and $\Pi_i = B_i^\top P - S^\top C_i$. Since $\sum_{i=1}^{N}\alpha_i = \sum_{j=1}^{N}\alpha_j = 1$, (23) can be rewritten as

$$\begin{bmatrix} \Psi_i & * \\ \sum_{i=1}^{N}\alpha_i B_i^\top P - S^\top\sum_{i=1}^{N}\alpha_i C_i & -R \end{bmatrix} \prec 0, \tag{24}$$

where Ψ_i is given by

$$He\{P\sum_{i=1}^{N}\alpha_i A_i\} + 2\gamma P - \sum_{i=1}^{N}\alpha_i C_i^\top Q\sum_{i=1}^{N}\alpha_i C_i.$$

The summation of matrices A_i, B_i, C_i are defined in (3), then (24) can be expressed as

$$\begin{bmatrix} He\{PA(\delta)\} + 2\gamma P - C(\delta)^\top QC(\delta) & * \\ B(\delta)^\top P - S^\top C(\delta) & -R \end{bmatrix} \prec 0. \tag{25}$$

Multiplying (25) by $[x^\top \;\; u^\top]$ on the left and by $[x^\top \;\; u^\top]^\top$ on the right, we obtain

$$\begin{aligned} &x^\top PA(\delta)x + x^\top A^\top(\delta)Px + x^\top 2\gamma Px + x^\top PB(\delta)u \\ &+ u^\top B(\delta)^\top Px - x^\top C(\delta)^\top QC(\delta)x - x^\top C(\delta)^\top Su \\ &- u^\top S^\top C(\delta)x - u^\top Ru < 0, \end{aligned} \tag{26}$$

as $y = C(\delta)x$ and $V = x^\top Px$, (26) can be rewritten as

$$\begin{aligned} &\nabla V^\top[A(\delta)x + B(\delta)u] + 2\gamma V \\ &-y^\top Qy - 2y^\top Su - u^\top Ru < 0. \end{aligned} \tag{27}$$

From (17), condition (27) implies that the system (1) is strictly QSR-dissipative for all $\delta \in \mathcal{D}$, completing the proof of item (i).

In addition, the control input u is a static output feedback given by the following equation

$$u = -R^{-1}S^\top y, \tag{28}$$

by substitution of (28) into (27), we obtain

$$\dot{V} + 2\gamma V < -y^\top\Delta y, \tag{29}$$

where $\Delta = SR^{-1}S^\top - Q$. Then, $\Delta \succeq 0$ is a sufficient condition for (9) to be satisfied, as follows

$$\dot{V} < -2\gamma V, \tag{30}$$

and system (1) is asymptotically stabilizable for all $\delta \in \mathcal{D}$ by the SOF (20) with a lower bound on the decay rate given by γ, completing the proof of all items.

4.1 Optimization Problem

Here, we formulate a linear SDP program in order to achieve $\Delta \succeq 0$. Firstly, from Madeira (2021) notice that $\Delta \preceq 0$ if

$$\begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \succeq 0, \tag{31}$$

as this is equivalent to

$$SR^{-1}S^\top - Q = \Delta \preceq 0, \tag{32}$$

if $R \succ 0$. On the other hand, our objective is achieve $\Delta \succeq 0$. Then, if we define a matrix M_d, such as

$$M_d = \begin{bmatrix} Q + \beta I & S \\ S^\top & R \end{bmatrix} \succeq 0, \tag{33}$$

with a scalar $\beta > 0$, we obtain the following condition

$$\Delta = SR^{-1}S^\top - Q \leq \beta I, \tag{34}$$

which means that $0 \leq \Delta \leq \beta I$ can be true. Then, by minimizing the function $tr(M_d)$, that is equivalent to minimize matrices Q and R, we might approach some positive value for the left side of condition (34), i.e. $\Delta \succeq 0$, as $tr(M_d) = 0 \Leftrightarrow M_d = 0$ (Yang, 1995).

4.2 SOF Design Algorithm

A systematic procedure for SOF design can be proposed.

(i) Consider an uncertain LTI system (1) with system matrices represented in the polytopic form (3).
(ii) Specify a lower bound on the decay rate $\gamma > 0$, some $\beta > 0$ for M_d in (33), and solve the following linear SDP program.

$$\begin{aligned} &\textit{minimize } tr(M_d), \\ &\textit{subject to } P \succ 0, \text{ (18), (19) and (33).} \end{aligned} \tag{35}$$

By applying this algorithm, we intend to find matrices (Q, S, R, P) that guarantee robust strict QSR-dissipativity of the system and, at the same time, fulfills $\Delta \succeq 0$, which guarantees the asymptotic stabilization by the SOF gain $K = R^{-1}S^\top$. Also, if we consider the system output matrix $C = I$, a static state feedback gain can be designed to stabilize system (1) by solving the same optimization problem (35). Moreover, it is important to highlight that the scalar $\beta > 0$ can be chosen as small a value, typically smaller than 1, without loss of generality (Madeira, D. de S., 2021). In many cases, $\beta = 0.1$ is a good choice, however if $\Delta \succeq 0$ is not achieved with this initial guess, a line search can be done.

5. NUMERICAL EXAMPLES

In this section, we present simulation results of our strategy by applying it to uncertain LTI systems. For the implementation we use conventional SDP tools provided by Lofberg (2004) and Sturm (1999).

5.1 Example 1

Consider the open-loop unstable system recently analyzed as the first numerical example of Behrouz et al. (2021), where the two vertices are given by the following matrices

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -0.9896 & 17.41 & 96.15 \\ 0.2648 & -0.8512 & -11.39 \\ 0 & 0 & -30 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -1.702 & 50.72 & 263.5 \\ 0.2201 & -1.418 & -31.99 \\ 0 & 0 & -30 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} -97.78 \\ 0 \\ 3 \end{bmatrix}, \; B_2 = \begin{bmatrix} -85.09 \\ 0 \\ 3 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{36}
$$

Applying the algorithm presented in section 4.2 we can determine matrices Q, S, R, P and a SOF gain that ensures closed-loop stability with a lower bound on the decay rate. For $\gamma = 1,\ \beta = 0.1$, and assuming $P \geq 10^{-2} I_n$, we obtain

$$
Q = \begin{bmatrix} 5.1449 & -0.0223 \\ -0.0223 & -0.0578 \end{bmatrix}, \; S = \begin{bmatrix} -4.8164 \\ 0.0448 \end{bmatrix},
$$

$$
R = 4.4371, \; P = \begin{bmatrix} 0.1719 & -0.1766 & 2.9419 \\ -0.1766 & 4.2030 & -5.5909 \\ 2.9419 & -5.5909 & 121.1968 \end{bmatrix} \succ 0,
$$

which leads to

$$
\Delta = \begin{bmatrix} 0.0833 & -0.0264 \\ -0.0264 & 0.0583 \end{bmatrix} \succ 0.
$$

Then the system is stabilizable and the SOF gain is given by

$$
K = [1.0855 \;\; -0.0101] .
$$

The two vertices of the system (36) are open loop unstable. The eigenvalues for the two vertices of the closed-loop system are $(-111.1108, -25.8358, -1.0343)$ and $(-106.1111, -14.2648, -5.1093)$, respectively. Figure 1 presents the closed-loop response of both vertices with $x(0) = [1 \;\; 1 \;\; 1]^\top$. The designed SOF stabilized the system, then the effectiveness of the strategy on the stabilization is verified. Behrouz et al. (2021) also proposed a strategy for SOF stabilization of uncertain LTI systems. While we consider a lower bound on the decay rate, they consider the constraint on the closed-loop pole location. Also, as in our work, they do not impose any restriction on the output matrix. However, Theorem 1 from Behrouz et al. (2021) employs more decision variables (d.v.) for the design of the SOF than the strategy presented herein, as shown in Table 1, thus being more numerically complex to solve.

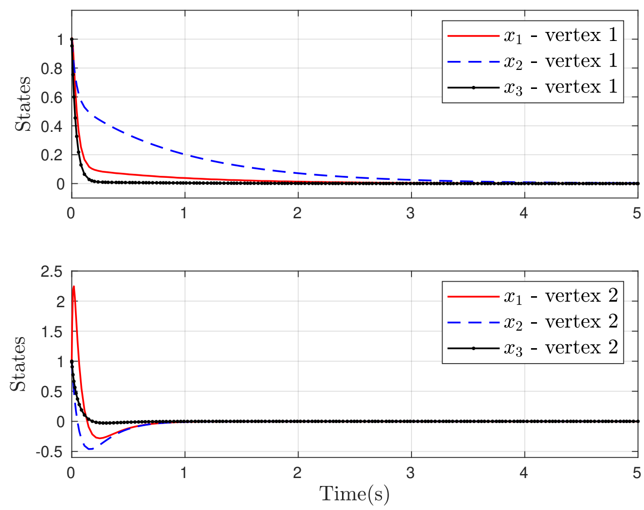

Figure 1. Example 1: States of the closed-loop response.

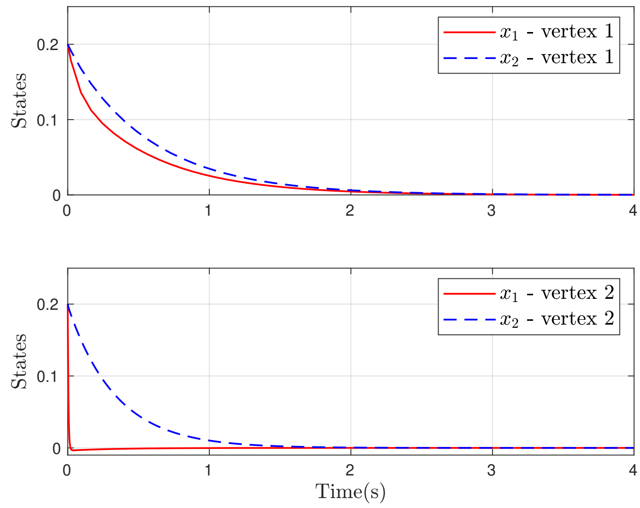

Figure 2. Example 2: States of the closed-loop response.

Table 1. Numerical complexity of SOF design for system (36)

	(Behrouz et al., 2021)	Proposed Approach
Nº of d.v.	18	12

5.2 Example 2

Consider the system analysed in Sereni et al. (2018) , where the two vertices are as follows

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -1 & 10 \\ -1 & -1 \end{bmatrix}, \; A_2 = \begin{bmatrix} a & -4 \\ -2 & -3 \end{bmatrix}, \; B_2 = \begin{bmatrix} b \\ 0 \end{bmatrix} \\
B_1 &= \begin{bmatrix} -9 \\ 0 \end{bmatrix}, \; C_1 = [1 \;\; 0], \; C_2 = C_1.
\end{aligned} \tag{37}
$$

Sereni et al. (2018) presented a feasibility analysis with the variation of coefficients $80 \leq a \leq 200$ and $-180 \leq b \leq -80$ of system (37), and a fixed decay rate $\gamma = 0.6$. Here, we choose $a = 80$ and $b = -180$ that is not in the feasibily region obtained in Sereni et al. (2018). Also, we selected a bigger decay rate $\gamma = 1$. Applying the algorithm presented in section 4.2 with $\beta = 0.1$ and assuming $P \geq 10^{-2} I_n$ as an initialization, we obtain

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$$Q = 20.7209,\ S = -12.9715,\ R = 8.0813,$$
$$P = \begin{bmatrix} 0.1000 & -0.0002 \\ -0.0002 & 0.9041 \end{bmatrix} \succ 0,$$

which leads to $\Delta = 0.1 \succ 0$, then the system is stabilizable and the SOF gain is given by

$$K = 1.6051.$$

The eigenvalues of the closed-loop system for the two vertices are $(-14.7171, -1.7290)$ and $(-208.9617, -2.9612)$, respectively. Simulation results for two vertices of the closed-loop system are presented in Figure 2, for initial conditions $x(0) = [0.2 \quad 0.2]^\top$. The closed-loop system is stable as expected.

6. CONCLUSION

A new strategy for static output feedback stabilization of uncertain linear systems with a lower bound on the decay rate has been proposed in this paper. We used the definition of strict QSR-dissipativity to formulate sufficient LMI conditions to solve the SOF control problem. Some numerical examples from the literature were used to demonstrate the effectiveness of the proposed strategy. The advantages of this approach are: i) it does not need to solve a state feedback control problem in a first stage, as it is common in the field, ii) in many cases it can be solved in a non-iterative manner, iii) no restriction on the output matrix is considered. Moreover, future work envisages the development of an iterative approach, to deal with the cases that we do not achieve $\Delta \succeq 0$, and also a detailed comparison with strategies already known in the literature as Felipe and Oliveira (2020); Agulhari et al. (2012); Shu and Lam (2009); Dong and Yang (2007). Finally, future works will also consider parameter dependent Lyapunov functions in order to decrease conservatism of conditions.

REFERENCES

Agulhari, C.M., Oliveira, R.C.L.F., and Peres, P.L.D. (2012). LMI relaxations for reduced-order robust H-infinity control of continuous-time uncertain linear systems. *IEEE Transactions on Automatic Control*, 57(6), 1532–1537.

Apkarian, P. and Noll, D. (2006). Nonsmooth $\mathcal{H}_\infty$ synthesis. *IEEE Transactions on Automatic Control*, 51(1), 71–86.

Astolfi, A., Ortega, R., and Sepulchre, R. (2002). Stabilization and disturbance attenuation of nonlinear systems using dissipativity theory. *Eur. J. Control*, 8(5), 408–431.

Behrouz, H., Mohammadzaman, I., and Mohammadi, A. (2021). Robust static output feedback $\mathcal{H}_2/\mathcal{H}_\infty$ control synthesis with pole placement constraints: An LMI approach. *International Journal of Control, Automation and Systems*, 19(1), 241–254.

Bernussou, J., Peres, P.L.D., and Geromel, J.C. (1989). A linear programming oriented procedure for quadratic stabilization of uncertain systems. *Systems & Control Letters*, 13(1), 65–72.

Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge University Press.

Brogliato, B., Lozano, R., Maschke, B., and Egeland, O. (2020). *Dissipative Systems Analysis and Control - Theory and Applications*. Springer-Verlag, London, UK.

Crusius, C.A.R. and Trofino, A. (1999). Sufficient LMI conditions for output feedback control problems. *IEEE Transactions on Automatic Control*, 44(5), 1053–1057.

Dong, J. and Yang, G.H. (2007). Static output feedback control synthesis for linear systems with time-invariant parametric uncertainties. *IEEE Transactions on Automatic Control*, 52(10), 1930–1936.

Felipe, A. and Oliveira, R.C.L.F. (2020). An LMI-based algorithm to compute robust stabilizing feedback gains directly as optimization variables. *IEEE Transactions on Automatic Control*, 1–1. doi:10.1109/TAC.2020.3038359.

Feng, Z., Lam, J., and Shu, Z. (2013). Dissipative control for linear systems by static output feedback. *International Journal of Systems Science*, 44(8), 1566–1576.

Gahinet, P. and Apkarian, P. (2011). Structured $\mathcal{H}_\infty$ synthesis in MATLAB. *IFAC Proceedings Volumes*, 44(1), 1435–1440.

Haddad, W.M. and Chellaboina, V. (2011). *Nonlinear dynamical systems and control: a Lyapunov-based approach*. Princeton University Press.

Hill, D. and Moylan, P. (1976). The stability of nonlinear dissipative systems. *IEEE Transactions on Automatic Control*, 21(5), 708–711.

Lofberg, J. (2004). YALMIP: A toolbox for modeling and optimization in MATLAB. In *2004 IEEE international conference on robotics and automation*, 284–289. IEEE.

Madeira, D. de S. (2021). Necessary and sufficient dissipativity-based conditions for feedback stabilization. *IEEE Transactions on Automatic Control*, doi: 10.1109/TAC.2021.3074850.

Madeira, D. de S. and Viana, V.V. (2020). An application of QSR-dissipativity to the problem of static output feedback robust stabilization of nonlinear systems. *Anais da Sociedade Brasileira de Automática*, 2(1).

Oliveira, R.C.L.F. and Peres, P.L.D. (2007). Parameter-dependent LMIs in robust analysis: Characterization of homogeneous polynomially parameter-dependent solutions via LMI relaxations. *IEEE Transactions on Automatic Control*, 52(7), 1334–1340.

Ortega, R. and Garcia-Canseco, E. (2004). Interconnection and damping assignment passivity-based control: A survey. *European Journal of control*, 10(5), 432–450.

Sadabadi, M.S. and Peaucelle, D. (2016). From static output feedback to structured robust static output feedback: A survey. *Annual reviews in control*, 42, 11–26.

Sereni, B., Manesco, R.M., Assunção, E., and Teixeira, M.C.M. (2018). Relaxed LMI conditions for the design of robust static output feedback controllers. *IFAC-PapersOnLine*, 51(25), 428–433.

Shishkin, S.L. and Hill, D.J. (1995). Dissipativity and global stabilizability of nonlinear systems. In *Proceedings of 1995 34th IEEE Conference on Decision and Control*, volume 3, 2227–2232. IEEE.

Shu, Z. and Lam, J. (2009). An augmented system approach to static output-feedback stabilization with $\mathcal{H}_\infty$ performance for continuous-time plants. *International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal*, 19(7), 768–785.

Sturm, J.F. (1999). Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones. *Optimization Methods and Software*, 11(1-4), 625–653.

Tanaka, K., Ikeda, T., and Wang, H.O. (1998). Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs. *IEEE Transactions on Fuzzy Systems*, 6(2), 250–265.

Veselý, V. (2001). Static output feedback controller design. *Kybernetika*, 2(37), 205–221.

Willems, J.L. (1972). A system theory approach to unified electrical machine analysis. *International Journal of Control*, 15(3), 401–418.

Yang, X.M. (1995). A generalization of a matrix trace inequality. *Journal of Mathematical Analysis and Applications*, 189(3), 897–900.