MULTIVARIABLE VIRTUAL REFERENCE FEEDBACK TUNING WITH BAYESIAN REGULARIZATION

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Abstract— This paper proposes the use of regularization on the multivariable formulation of the Virtual Reference Feedback Tuning (VRFT). When the process to be controlled has a significant amount of noise, the standard VRFT approach, that uses the instrumental variable technique, provides estimates with very poor statistical properties. To cope with that, this paper considers the use of regularization on the estimation procedure, reducing the covariance error at the cost of inserting a small bias. Also, this paper explains different types of regularization matrices and presents the methodology to tune these matrices. In order to demonstrate the benefits of the proposed formulation, a numerical example is presented.

Keywords— Control Theory and Applications, Data-driven Control, System Identification, Regularization.

1 Introduction

Data-driven control methods are a class of techniques used to design a controller without deriving a mathematical model of the process. Instead, it is assumed that the user can run experiments on the system and collect signals that will be used to adjust the controllers (Bazanella et al., 2011). Several techniques use this idea where the most notorious are: Iterative Feedback Tuning (IFT) (Hjalmarsson et al., 1998), Correlation based Tuning (CbT) (Karimi et al., 2004), Virtual Reference Feedback Tuning (VRFT) (Campi et al., 2002) and Optimal Controller Identification (OCI) (Campestrini et al., 2017). While the first two are iterative by nature, i.e. a sequence of experiments is used to iteratively refine the gains of the controller, the last two are non-iterative (direct), i.e. the controllers are obtained from just one set of experimental data.

Virtual Reference Feedback Tuning is probably the most used data-driven technique due to its simplicity. The method solves a convex optimization problem using only one or two set of experimental data. Moreover, under some ideal conditions the obtained controller is optimal. The method has been applied to many different applications from combustion motors, to chemical plants, and even to unmanned aerial vehicles. Also, several extensions have emerged in the literature allowing its use with non minimum phase (Campestrini et al., 2011) and MIMO systems (Campestrini et al., 2016). However, it is widely known that under non-ideal conditions, for instance measurement noise on the signals, the controller parameter estimate presents a large variance and that it often results on an unstable closed loop system.

In order to mitigate some problems of the single-input single-output (SISO) VRFT method, the article Rallo et al. (2016) proposes to add a regularization term to the objective function, resulting still in a convex problem that may produces biased estimates to the controller parameters but with a lower variance. The article presents a *Bayesian* perspective to the problem using similar ideas of Pillonetto and De Nicolao (2010) and Chen et al. (2012). A priori information about the parameters is estimated, assuming that they are Gaussian distributed with zero mean, and incorporated to the regularization procedure.

In this work we propose to use the same ideas presented in Rallo et al. (2016) to extend the MIMO VRFT technique to include a regularization term. We also give a *Bayesian* perspective to the problem and compare four different parameterizations to the regularization term. The proposed technique is evaluated in a numerical example where it's shown that the regularization term improves the performance of the closed loop system, improves the stability of the method and reduces the variance of the obtained controllers.

The paper is organized as follow. Section 2 presents the preliminaries describing properties of the system, controller and desired closed loop performance. Section 3 describes the state of the art MIMO VRFT method, while Section 4 presents the proposed modification including the regularization term and its Bayesian perspective. A numerical example is given in Section 5 and the article finishes in Section 6 with a brief conclusion.

2 Preliminaries

Consider a MIMO discrete-time linear time-invariant (LTI) system:

$$y(t) = G_0(q)u(t) + H_0(q)e(t), \tag{1}$$

where q is the forward-shift operator, $y(t) \in \mathbb{R}^n$ is the system's output vector, $u(t) \in \mathbb{R}^n$ is the input vector, $G_0(q)$ is the process transfer function matrix, $H_0(q)$ the noise transfer function matrix and $e(t) \in \mathbb{R}^n$ a white noise vector, with zero mean and covariance matrix $E[e(t)e(s)^T] = \Sigma = diag(\sigma_{e1}^2, \sigma_{e2}^2, ..., \sigma_{en}^2)$. The matrices $G_0(q)$ and $H_0(q)$ are $n \times n$ and their elements are proper rational transfer functions.

In order to achieve some desired performance, e.g. reference tracking, disturbance rejection, the system's input vector is manipulated through a controller, more specifically, in this work, a LTI controller which belongs to a fixed user-defined controller class $\mathcal{C}$. The control signal $u(t)$ can then be written as

$$u(t) = C(q, P)(r(t) - y(t)), \tag{2}$$

where $r(t) \in \mathbb{R}^n$ is the reference vector and $C(q, P)$ is the $n \times n$ controller transfer function matrix, parametrized through the vector $P \in \mathbb{R}^p$. The controller class $\mathcal{C}$ is defined by

$$\mathcal{C} = \{C(q, P) : P \in \Omega \subseteq \mathbb{R}^p\}, \tag{3}$$

and its structure is given by

$$C(q, P) = \begin{bmatrix} C_{11}(q, \rho_{11}) & \dots & C_{1n}(q, \rho_{1n}) \\ \vdots & \ddots & \vdots \\ C_{n1}(q, \rho_{n1}) & \dots & C_{nn}(q, \rho_{nn}) \end{bmatrix}, \tag{4}$$

where $P = [\rho_{11}^T \; \rho_{12}^T \; \dots \; \rho_{1n}^T \; \dots \; \rho_{nn}^T]^T$. It is assumed that each subcontroller has a linear parametrization, i.e. they can be written as

$$C_{ij}(q, \rho_{ij}) = \rho_{ij}^T \overline{C}_{ij}(q), \rho_{ij} \in \mathbb{R}^{p_{ij}}, \tag{5}$$

with $\overline{C}_{ij}(q)$ being a $p_{ij}-$vector of fixed proper rational functions. Observe that, for instance, PI and PID controllers can be described by (5). Notice that each subcontroller may have a different structure, provided that they are linear in the parameters.

Considering (1-2) the closed loop system becomes:

$$y(t, P) = T(q, P)r(t) + S(q, P)v(t), \tag{6}$$

with

$$v(t) = H_0(q)e(t) \tag{7}$$

$$S(q, P) = (G_0(q)C(q, P) + I)^{-1} \tag{8}$$

$$T(q, P) = S(q, P)G_0(q)C(q, P). \tag{9}$$

It is evident, then, the dependence of the system's output on the parameter vector P. Therefore, this work deals with the problem of tuning P to reach good closed loop performance considering a Model Reference framework, where the desired closed loop behavior is specified through a transfer function, namely the *reference model* and denoted by $T_d(q)$. So, the parameter vector P could be obtained by solving the optimization problem:

$$\hat{P} = \arg\min_P J_{MR}(P) \tag{10}$$

$$J_{MR}(P) = \sum_{t=1}^{N} ||(T_d(q) - T(q, P))r(t)||_2^2, \tag{11}$$

which intends to minimize the difference between the desired closed loop behavior and the one achieved with $C(q, P)$. However, the objective function $J_{MR}(P)$ is nonconvex and the solution of the optimization problem is difficult to be achieved.

The controller that leads the system to match exactly the desired performance is known as the *ideal controller* $C_d(q)$:

$$C_d(q) = G_0(q)^{-1}T_d(q)(I - T_d(q))^{-1}. \tag{12}$$

which can only be calculated if the model $G_0(q)$ of the process is known. It's important to emphasize that if $C_d(q) \in \mathcal{C}$ then $\exists\, P_0 \;:\; C(q, P_0) = C_d(q)$.

In this article it is assumed that the user has access to input and output data from the process but the model $G_0(q)$ is unknown. Using only data from the experiment, the user desires to obtain the gains of the controller that minimize the objective criterion $J_{MR}(P)$.

3 The multivariable VRFT

The VRFT method is a one-shot data-driven control design technique that aims to solve the problem described in (10) using only input and output data measured from the system, without the need of a mathematical model. Its original formulation was developed for the SISO LTI case in Campi et al. (2002). The method was extended for MIMO systems in Nakamoto (2004) and a more detailed analysis was given in Formentin et al. (2012). However, these MIMO extensions had a significant drawback: the same reference model had to be chosen for each loop, in order to provide an unbiased parameter estimate. In Campestrini et al. (2016) a new formulation was developed, allowing the user to choose a wider class of reference models. This work aims to enhance the statistical properties of the latter VRFT formulation using regularization techniques. Also, it is worth mentioning that the standard VRFT approaches and also the new formulation proposed here can be applied even to unstable processes.

The main idea of the VRFT method is to rewrite the optimization problem described in (10) as

$$\hat{P} = \arg\min_{P} J_{VR}(P) \tag{13}$$

$$J_{VR}(P) = \sum_{t=1}^{N} ||F(q)[u(t) - C(q,P)\overline{e}(t)]||_2^2, \tag{14}$$

where $F(q)$ is a prefilter (used as an additional degree of freedom), $u(t)$ is the input signal measured from the system and $\overline{e}(t)$ is the virtual error, defined as $\overline{e}(t) = (T_d(q)^{-1} - I)y(t)$, with $y(t)$ being the measured output signal. Since the controller has a linear parametrization, $J_{VR}(P)$ is convex and the optimization problem has a trivial solution, that can be reached by the least squares algorithm.

Under ideal conditions, where there is no noise on the system and the ideal controller $C_d(q)$ belongs to the user-specified controller class $\mathcal{C}$, then P_0 is the solution of (13). If the ideal controller doesn't belong to $\mathcal{C}$, then the minimum of $J_{VR}(P)$ and $J_{MR}(P)$ doesn't coincide. In this case, the filter $F(q)$ is designed to approximate the minimum of both cost functions. A suitable practical choice of the filter is given by $F(q) = T_d(q)(I - T_d(q))$. For more detailed reasoning about the filter design, see Campestrini et al. (2016).

On the other hand, if the system is corrupted by noise, the estimates given by the least squares algorithm for the problem (13) will be biased, even if $C_d(q) \in \mathcal{C}$ (Campestrini et al., 2016). To cope with that inconvenience, *basic instrumental variables* are used. Therefore, the parameters are computed as

$$\hat{P}_{iv} = \text{Sol}\left\{\frac{1}{N}\sum_{t=1}^{N}\zeta_F(t)[\varphi_F^T(t)P - u_F(t)] = 0\right\} \tag{15}$$

$$\hat{P}_{iv} = \left(\frac{1}{N}\sum_{t=1}^{N}\zeta_F(t)\varphi_F^T(t)\right)^{-1}\left(\frac{1}{N}\sum_{t=1}^{N}\zeta_F(t)u_F(t)\right), \tag{16}$$

with

$$u_F(t) = F(q)u(t), \quad \varphi_F = [A_1(t)\dots A_n(t)], \tag{17}$$

$$A_x(t) = \begin{bmatrix} F_{x1}(q)E_1(t) \\ \vdots \\ F_{xn}(q)E_n(t) \end{bmatrix}, \tag{18}$$

$$E_x(t) = \begin{bmatrix} \overline{C}_{x1}(q)\overline{e}_1(t) \\ \vdots \\ \overline{C}_{xn}(q)\overline{e}_n(t) \end{bmatrix}, \tag{19}$$

and $\zeta_F(t)$ being the instrumental variable. The instrumental variable can be generated with data collected from a second experiment on the system, with the same input sequence $u(t)$. Let $y'(t)$ be the output of the second experiment, then $\zeta_F(t) = [A'_1(t)\dots A'_n(t)]$ is built similarly as $\varphi_F(t)$, but with $\overline{e}'(t) = (T_d(q)^{-1} - I)y'(t)$ in (19). In this case, $\hat{P}_{iv}$ is an unbiased estimate of P_0, but as the noted in (Campestrini et al., 2016) the estimate has a large variance. In this work we introduce a regularization term to the objective function aiming to reduce the error between the estimate and the ideal controller.

4 Multivariable VRFT with Bayesian regularization

It is a well-known fact that the instrumental variable estimate has a large covariance, which can't reach the Cramer-Rao lower bound (Ljung, 1999). When the variance of the noise is large, the estimates obtained with classical VRFT formulation may lead to closed loop behavior very different from the reference model. In Formentin and Karimi (2014) and Rallo et al. (2016) modifications to the SISO VRFT were proposed to include a regularization term on the objective criterion to reduce the error between the estimate and the ideal controller. The same idea is extended here, where the main technical challenge is the application of this technique to MIMO systems, that presents more complex control structure and also lead to a different Bayesian interpretation. To do so, it is necessary to add a regularization term on the objective function (14):

$$\hat{P}_{REG} = \arg\min_{P} J_{VR}^{REG}(P) \tag{20}$$

$$J_{VR}^{REG}(P) = J_{VR}(P) + P^T D P, \tag{21}$$

with $D \in \mathbb{R}^{p\times p}$ being positive semi-definite, also called the regularization matrix. Analogously to what was done before, using the basic instrumental variable, the new regularized estimate is given by

$$\hat{P}_{iv}^{reg} = \text{Sol}\left\{\frac{1}{N}\sum_{t=1}^{N}\zeta_F(t)[\varphi_F^T(t)P - u_F(t)] + DP = 0\right\} \tag{22}$$

$$\hat{P}_{iv}^{reg} = \left(\frac{1}{N}\sum_{t=1}^{N}\zeta_F(t)\varphi_F^T(t) + D\right)^{-1}\left(\frac{1}{N}\sum_{t=1}^{N}\zeta_F(t)u_F(t)\right), \tag{23}$$

with $\varphi_F(t)$, $\zeta_F(t)$ and $u_F(t)$ as defined previously.

Equation (23) provides a new estimate of P for the MIMO VRFT considering regularization which is expected to provide biased estimates, but with much smaller variance (Chen et al., 2012). The problem now is to choose the matrix D in order to reduce the estimate error. A Bayesian perspective of the identification procedure will be described in the next subsection, using the same ideas developed on the System Identification community to tune D.

4.1 The Bayesian perspective

The Bayesian perspective of the identification procedure provides an alternative point of view and new insights to regularization. In the Bayesian perspective, the parameter P itself is considered a random variable with prior Gaussian distribution with zero mean and covariance matrix Π (also called *kernel*), i.e. $P \sim \mathcal{N}(0, \Pi)$. It has been shown, in recent literature (Chen et al., 2012), that this interpretation yields the same estimate that the regularized one, if the matrix D is written as

$$D_i = \sigma_{ei}^2 \Pi_i^{-1}, \tag{24}$$

where

$$D = \begin{bmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_n \end{bmatrix}, \tag{25}$$

$$\Pi = \begin{bmatrix} \Pi_1 & 0 & \dots & 0 \\ 0 & \Pi_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Pi_n \end{bmatrix}, \tag{26}$$

and Π_i is related to the i-th output, which means it has dimension $p_i \times p_i$, with $p_i = \sum_{j=1}^{n} p_{ij}$. This fact restates that regularization is closely related to prior estimates (Chen et al., 2012).

Still, it is necessary to estimate the covariance matrices Π_i. Let them be unknown and parametrized by the vector of *hyper-parameters* $\eta = [\eta_{11}^T \ \eta_{12}^T \ \dots \ \eta_{1n}^T \ \dots \eta_{nn}^T]^T$. Also, assume that the responses from different inputs have no mutual correlation. So, it is natural to partition Π_i as (Pillonetto et al., 2014)

$$\Pi_i(\eta_i) = \begin{bmatrix} \Pi_{i1}(\eta_{i1}) & 0 & \dots & \dots & 0 \\ 0 & \ddots & 0 & \dots & 0 \\ \vdots & 0 & \Pi_{ij}(\eta_{ij}) & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \dots & 0 & \Pi_{in}(\eta_{in}) \end{bmatrix} \tag{27}$$

where j represents the jth input. Each matrix Π_{ij} has dimension $p_{ij} \times p_{ij}$.

Now, let us discuss about the structure of the matrices Π_{ij}. Let it reflect the size and correlation of the controller's impulse response coefficients (Chen et al., 2012). If the optimal controller is stable, then the variance of P tends to zero exponentially and it possesses a certain size λ and a decay rate α. So, the simpler choice would be to set Π_{ij} diagonal with the (k,k)-th element being

$$\Pi_{ij}(\eta_{ij}, k) = \lambda_{ij} \alpha_{ij}^k, \tag{28}$$

with $\lambda_{ij} \geq 0$, $0 \leq \alpha_{ij} < 1$ and $\eta_{ij} = [\lambda_{ij} \ \alpha_{ij}]^T$. This is also known as the *Diagonal* (DI) kernel parametrization (Chen et al., 2012). Yet, more information about the controller's impulse response could be considered. Suppose that the controller's impulse response is smooth. Then, neighboring values of the kernel matrix should have positive and high correlation. That being said, a suitable choice for Π_{ij} should be a matrix whose (k,l) elements are

$$\Pi_{ij}(\eta_{ij}, k, l) = \lambda_{ij} \alpha_{ij}^{(k+l)/2} \mu_{ij}^{|l-k|}, \tag{29}$$

with $\lambda_{ij} \geq 0$, $0 \leq \alpha_{ij} < 1$ and $|\mu_{ij}| \leq 1$, $\eta_{ij} = [\lambda_{ij} \ \alpha_{ij} \ \mu_{ij}]^T$. This kind of parametrization is also called *Diagonal/Correlated* (DC) (Chen et al., 2012). Another type of parametrization, described in (Chen et al., 2012), is a special case of the DC kernel, where $\mu = \sqrt{\alpha}$, which gives the *Tuned/Correlated* (TC) kernel:

$$\Pi_{ij}(\eta_{ij}, k, l) = \lambda_{ij} \alpha_{ij}^{\max(k,l)}, \tag{30}$$

with $\lambda_{ij} \geq 0$, $0 \leq \alpha_{ij} < 1$, $\eta_{ij} = [\lambda_{ij} \ \alpha_{ij}]^T$. A fourth type of kernel, frequently used and developed in the context of *Gaussian Process Regression* (GPR), is known as the *Stable Spline* (SS) kernel, presented in Pillonetto and De Nicolao (2010):

$$\Pi_{ij}(\eta_{ij}, k, l) = \lambda_{ij} \left(\frac{\alpha_{ij}^{k+l+\max(k,l)}}{2} - \frac{\alpha_{ij}^{3\max(k,l)}}{6} \right), \tag{31}$$

with $\lambda_{ij} \geq 0$, $0 \leq \alpha_{ij} < 1$, $\eta_{ij} = [\lambda_{ij} \ \alpha_{ij}]^T$.

Now that different types of kernel parametrizations have been discussed, the next step should be the estimation of η and Σ (if the latter is unknown). The most common technique for this purpose is the *Empirical Bayes* approach, where the hyper-parameters are estimated by the maximum likelihood approach, given the observations and its a priori distribution. More details about this technique are argued on Chen et al. (2012) and Pillonetto et al. (2014).

In this work it is proposed the comparison of these four (DI, DC, TC and SS) parametrizations to matrix D in order to evaluate their use with the MIMO VRFT technique.

5 A numerical example

To illustrate the efficiency of the proposed methodology, a numerical example was developed. Consider the following process, also used in Campestrini et al. (2016):

$$G_0(q) = \begin{bmatrix} \frac{0.09516}{(q-0.9048)} & \frac{0.03807}{(q-0.9048)} \\ \frac{-0.02974}{(q-0.9048)} & \frac{0.04758}{(q-0.9048)} \end{bmatrix}, \tag{32}$$

$H_0(q) = I$, and the reference model was chosen as an uncoupled transfer function:

$$T_d(q) = \begin{bmatrix} \frac{0.25}{q-0.75} & 0 \\ 0 & \frac{0.4}{q-0.6} \end{bmatrix}, \tag{33}$$

that ensures null steady-state error for step reference signals. The ideal controller is given by

$$C_d(q) = \begin{bmatrix} \frac{2.102q-1.902}{q-1} & \frac{-2.69q+2.434}{q-1} \\ \frac{1.314q-1.189}{q-1} & \frac{6.725q-6.085}{q-1} \end{bmatrix}, \quad (34)$$

which is a full PI controller. The controller class $\mathcal{C}$ was chosen also as a full PI controller, so we got

$$P_0 = [\rho_{11_0}^T \ \rho_{12_0}^T \ \rho_{21_0}^T \ \rho_{22_0}^T]^T \quad (35)$$

$$\rho_{11_0} = [2.012 \ \ -1.902]^T \quad (36)$$

$$\rho_{12_0} = [-2.69 \ 2.343]^T \quad (37)$$

$$\rho_{21_0} = [1.314 \ \ -1.189]^T \quad (38)$$

$$\rho_{22_0} = [6.725 \ \ -6.085]^T. \quad (39)$$

Two open loop experiments were simulated to collect the data and the instrumental variable, required to identify the controller. The input signals were defined as *Pseudo-Random Binary Sequences* (PRBS) with size $N = 300$. The noise variance was specified to yield a *Signal-to-Noise Ratio* (SNR) of 1.5 for each output. The filter $F(q)$ was computed as $T_d(q)(I - T_d(q))$ as mentioned beforehand.

Concerning the tuning of the different types of kernel matrices $\Pi(\eta)$ and the estimation of the noise variances to determine D as in (24), the MATLAB function *arxRegul* was employed. As a matter of fact, this function implements an algorithm based on the classical system identification scenario (with additive noise on the outputs), which is not the case here: the controller identification is an *errors-in-variables* problem (the noise is actually on the inputs) (Söderström, 2007). Despite that difference of characteristics, as demonstrated in Rallo et al. (2016), the MATLAB function also allows a good kernel estimation.

To evaluate the proposed technique, 100 Monte Carlo simulations were run with distinct noise realizations. At each run, input and output data were collected and the controller gains were estimated using the classical MIMO VRFT via (16) and the regularized one via (23) considering the four parameterizations of matrix D. Three different comparison were evaluated: the objective criterion $J_{MR}(P)$, stability of the closed loop system with the obtained controller and MSE error of the parameters.

5.1 Objective criterion

The main objective of introducing regularization to the VRFT method was to achieve better closed loop performance. The performance evaluation was done through the objective function $J_{MR}(P)$. To do so, the signal $r(t)$ was defined as two unitary step reference signals: $r_1(t)$ occurring at $t = 1$ and $r_2(t)$ occurring at $t = 101$ with a total of 200 samples. Figure 1 exhibits the *boxplots* of $J_{MR}(\hat{P})$ obtained on the Monte Carlo runs for each VRFT

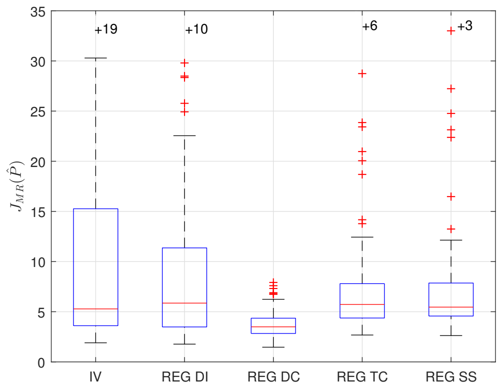

Figure 1: Comparison of $J_{MR}(P)$ for the classical and the regularized VRFT with different choices of kernel.

approach (classical and regularized with different choices of kernel parametrization).

Analyzing the boxplots, it's clear that the regularized approach overcame the classical one with respect to variance. Another point to be emphasized is that, among the regularized methods, the one with the Diagonal kernel presented the worst performance. This outcome is coherent, since this is the simpler structure of kernel parametrization. On the other hand, the DC kernel presented best performance regarding this criterion. This means that the estimation of its hyper-parameters was quite precise and the information of the a priori distribution collaborated to the estimation of P.

5.2 Stability of the closed loop system

To give an idea of what happens to the closed loop responses, a comparison of $T_d(q)r(t)$ and $T(q, \hat{P})r(t)$ is also displayed for the 100 Monte Carlo runs. The comparison was built for the worst (classical VRFT) and the best (regularized VRFT with DC kernel) scenarios attained. The signal $r(t)$ employed on these simulations was the same used before. Figure 2 demonstrates the closed loop responses for the classical VRFT, while Figure 3 demonstrates the closed loop responses for the regularized VRFT with the DC kernel. The desired closed loop responses are represented by black lines. Blue lines represent the responses of resulting stable systems and red lines represent responses of resulting unstable closed loop systems.

Figure 2 shows that the high variance of the IV estimate can result on unstable systems in some situations. In the case presented here, 18% of the resulting closed loop systems were unstable. Figure 3, still, shows smaller variance of the responses and that none of the resulting closed loop systems were unstable. It is important to highlight that this result was achieved even without

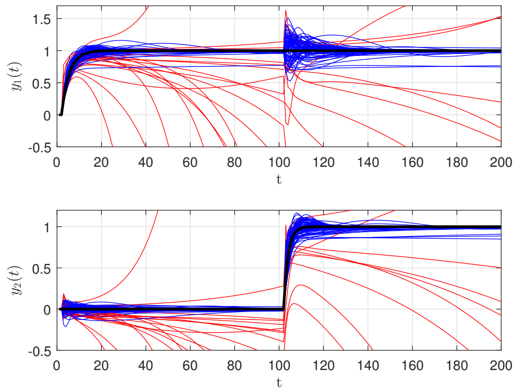

Figure 2: Comparison of $T_d(q)r(t)$ (black lines) and $T(q,\hat{P})r(t)$ (blue lines = stable systems, red lines = unstable systems) for the classical VRFT.

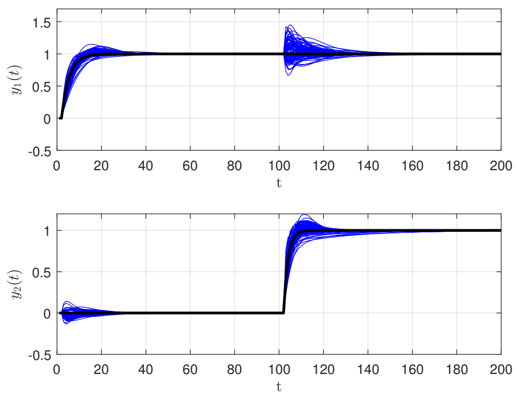

Figure 3: Comparison of $T_d(q)r(t)$ (black lines) and $T(q,\hat{P})r(t)$ (blue lines = stable systems) for the regularized VRFT with DC kernel.

using a stability constraint on the VRFT method. Such stability constraint can be seen at Van Heusden et al. (2011). This reinforces that introducing regularization to the VRFT methodology improved its statistical properties.

5.3 MSE of the estimates

Besides the analysis on the closed loop performance, it's also interesting to observe the effect of regularization on the parameter estimate $\hat{P}$. In order to examine that, the 8-dimension ellipsoid of 95% confidence interval was calculated for the samples. Then, to turn it visible in some matter, projections of the ellipsoid were drawn on the planes $\rho_{11}^0 \times \rho_{11}^1$ [1], $\rho_{12}^0 \times \rho_{12}^1$, $\rho_{21}^0 \times \rho_{21}^1$ and $\rho_{22}^0 \times \rho_{22}^1$. Figures 4, 5, 6 and 7 exhibit these projections, also for the best and the worst scenarios attained.

It can be seen that the estimates given by the classical VRFT approach were approximately unbiased, except at the projection on $\rho_{22}^0 \times \rho_{22}^1$.

[1] $\rho_{ij} = [\rho_{ij}^0 \ \rho_{ij}^1]^T$ since $\mathcal{C}$ is a full PI controller.

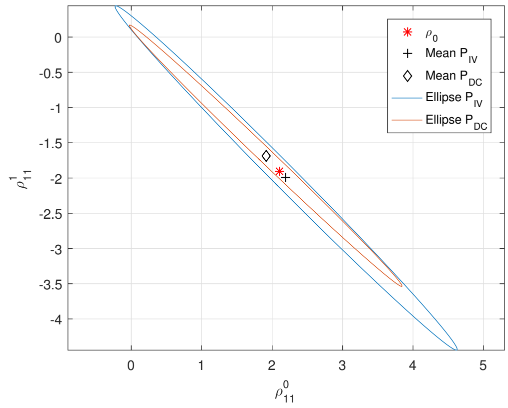

Figure 4: Projection of the 95% confidence ellipsoid on the plane $\rho_{11}^0 \times \rho_{11}^1$.

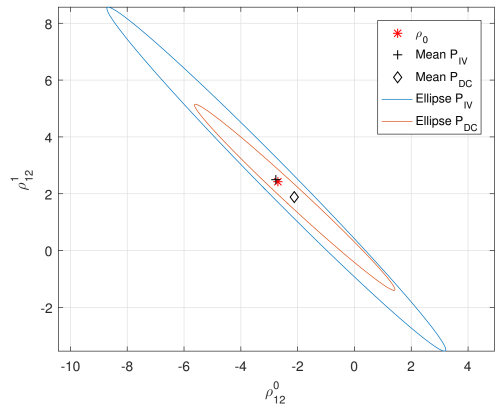

Figure 5: Projection of the 95% confidence ellipsoid on the plane $\rho_{12}^0 \times \rho_{12}^1$.

These bias that appears on the estimates can be explained by the limited amount of data available and the limited number of Monte Carlo runs. Moreover, the variance of the estimates, associated to the size of the ellipses, are really large. Yet, the estimates given by the regularized VRFT with the DC kernel presented bias as expected, but a considerable smaller variance. This fact, after all, improved the properties of the $J_{MR}(P)$ criterion and, accordingly, the closed loop performances.

Finally, other criterion that can be considered concerning the quality of the estimates $\hat{P}$ is the size of the MSE matrix. For each method, the MSE was estimated based on the Monte Carlo samples as

$$\widehat{MSE} = \frac{1}{100}\sum_{i=1}^{100}(\hat{P}-P_0)(\hat{P}-P_0)^T. \qquad (40)$$

The size of the matrices was measured by some of their fundamental quantities, as the maximum eigenvalue (max λ), the minimum eigenvalue (min λ), the trace and the determinant. Table 1 demonstrates these quantities, calculated for each

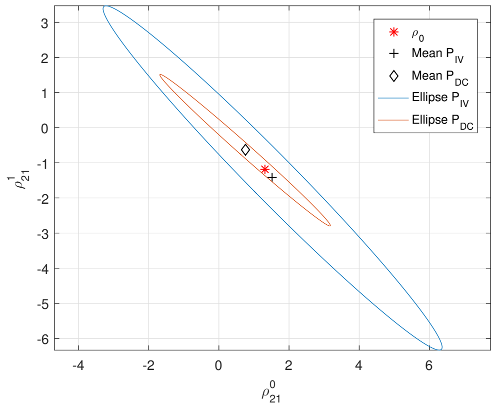

Figure 6: Projection of the 95% confidence ellipsoid on the plane $\rho_{21}^0 \times \rho_{21}^1$.

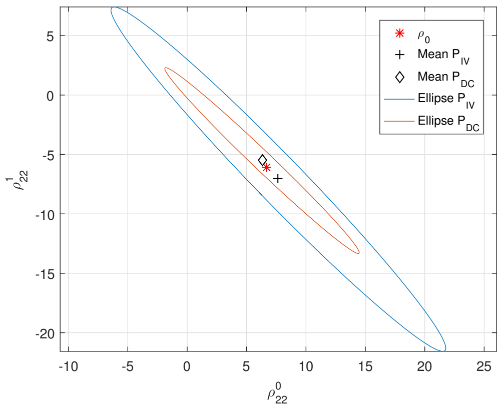

Figure 7: Projection of the 95% confidence ellipsoid on the plane $\rho_{22}^0 \times \rho_{22}^1$.

method here discussed. In general, the MSE quantities computed for the classical VRFT presented larger values. Furthermore, the DC based regularization, the one that held the best results for the $J_{MR}(P)$ criterion, also gave the smaller MSE values.

6 Conclusion

This work has proposed an extension to the MIMO VRFT method to include a regularization term with objective to reduce the variance introduced by the instrumental variables technique. A Bayesian perspective was presented and four different parameterizations for the D matrix were evaluated. A numerical example compared the classical MIMO VRFT with the proposed regularized and it showed that the use of regularization introduces a bias in the parameters estimation but reduces the variance, resulting in much better closed loop performance. The example shown that the DC regularization reduces the $J_{MR}(P)$ criterion, improves the stability of the closed loop system and reduces the MSE between the obtained controller and the ideal one.

Table 1: Quantities of $\widehat{MSE}$ matrices for each method

Type	max λ	min λ	Trace	Determinant
IV	28.21	15.61×10^{-4}	36.87	3.76×10^{-5}
DI	11.90	11.74×10^{-4}	14.54	4.26×10^{-9}
DC	8.95	6.34×10^{-4}	12.78	3.52×10^{-9}
TC	14.27	16.74×10^{-4}	18.04	4.71×10^{-8}
SS	14.23	11.44×10^{-4}	22.56	4.17×10^{-8}

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