DESIGN OF AN ADAPTIVE CONTROLLER TO IMPROVE THE CONDITION NUMBER OF THE INERTIA MATRIX OF SERIAL MANIPULATORS

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Abstract— The ill-conditioning of the inertia matrix of serial manipulators is a problem intrinsic to multi-link open serial chains, which may potentially reduce the accuracy and performance of most motion control techniques based on the robot's dynamic model. In more extreme cases, the ill-conditioning can even result in unstable behavior. In order to solve this, an adaptive control law is applied to the robot to improve the matrix conditioning while ensuring that the well-conditioned inertia matrix is positive definite and hence continues to have physical meaning. Simulation results on a serial manipulator with seven degrees of freedom show that the proposed control law outperforms other commonly used techniques in terms of smoother behavior, smaller steady-state error, and smaller condition number of the inertia matrix.

Keywords— Robot manipulator, adaptive control, dynamics, inertia matrix, ill-conditioning.

Resumo— O mau condicionamento da matriz de inércia de manipuladores seriais é um problema intrínseco às cadeias seriais abertas com múltiplos elos, o que pode potencialmente reduzir a precisão e desempenho da maioria das técnicas de controle de movimento. Em casos mais extremos, o mau condicionamento pode inclusive resultar em comportamentos instáveis, Para solucioná-lo, uma lei de controle adaptativo é aplicada ao robô para melhorar o condicionamento da matriz e ao mesmo tempo garantir que a matriz de inércia bem condicionada seja positiva definida e portanto continue a ter um significado físico. Os resultados de simulação em um manipulador serial com sete graus de liberdade mostram que a lei de controle proposta possui um desempenho melhor do que outras técnicas usualmente utilizadas em termos de comportamento mais suave, menor erro em estado estacionário e menor número de condicionamento da matriz de inércia.

Palavras-chave— Robô manipulador, controle adaptativo, dinâmica, matriz de inércia, mau condicionamento.

1 Introduction

The joint space inertia matrix (JSIM) of a robot manipulator plays an important role in the analysis and control of the robot's dynamic behavior. More specifically, the JSIM is specially important when dealing with forward dynamics, which is essential for simulation (Featherstone 2004, Shah et al. 2017), and when designing motion controllers based on the Euler-Lagrange equations (Siciliano et al. 2009). Although it is well known that the JSIM is positive definite independently of the robot configuration, this property does not guarantee the good conditioning of the matrix (Shen & Featherstone 2003). In a multi-link open serial chain, the links are connected to each other in a way that the penultimate link carries the last one, the antepenultimate link carries the last two and so on, and the base link carries all the others. As a result, the equivalent inertia of the links are extremely disparate, and this difference increases with the number of links, even if they are identical to each other (Agarwal et al. 2014), which leads to the ill-conditioning of the JSIM. Moreover, if the links are not all the same size, the condition number can be still higher (Featherstone 2004).

The ill-conditioning of the JSIM affects both the accuracy of simulation results and the control performance (Shen & Featherstone 2003). When the JSIM becomes ill-conditioned, small perturbations in the system can produce large changes in the numerical solutions (Agarwal et al. 2014). According to Featherstone (2004), this property is not just a numerical problem; rather it is intrinsic to a phenomenon of ill-conditioning in the mechanism itself, which suggests that the mechanism may be more difficult to control even if the JSIM is not used directly in the control input calculation.

Despite the fact that the JSIM's ill-conditioning is intrinsic to serial kinematic chains, its effects can be mitigated and therefore the control performance can be enhanced. One way to improve the JSIM's condition number is to add a well-conditioned positive definite matrix to it in the Euler-Lagrange equations. Such matrix could be related to the inertia of the actuators, instead of just adding an arbitrary matrix, in order to prevent the introduction of unnecessary inaccuracies to the model. However, if the added actuators' inertia matrix does not correspond to the actual one, the closed-loop system may still present steady-state error, which appears when introducing any inaccuracies in the robot model and these uncertainties are neither eliminated nor compensated by the control law. Adding an integrator to the control law is not sufficient to solve the steady-state error problem because, even though the actuators' inertia matrix is usually constant (Shen & Featherstone 2003), the disturbance depends on the robot acceleration, therefore it is time varying. More specifically, if the actual robot's inertia matrix is given by $\bar{\boldsymbol{M}} = \boldsymbol{M} + \boldsymbol{M}_m$, where $\boldsymbol{M}$ is the nominal robot's inertia matrix and $\boldsymbol{M}_m$ is the actuators' inertia matrix, the actual dynamic model is given by

$$\boldsymbol{\tau} = \bar{\boldsymbol{M}}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{w},$$

where $\boldsymbol{w} = \boldsymbol{M}_m\ddot{\boldsymbol{q}}$ is the time-varying disturbance.

An alternative solution to improve the JSIM's conditioning is to use an adaptive controller to compensate for the unknown inertia of the actuators. The basic idea is to estimate the uncertain parameters on-line based on the measured system signals, and use those estimates in the control input computation (Slotine & Li 1991). This would solve not only the problem of ill-conditioning, but also any uncertainties related to lack of information or changes in the robot's dynamics produced by interaction with the environment. Cheah et al. (2006*a*) proposed an adaptive controller for robots with uncertain kinematics and dynamics—in addition to adaptation to actuator parameters—and showed that, although there is no guarantee that the estimated parameters converge to the real ones, the closed-loop system is asymptotic stable, which is our goal.

Some adaptive controllers require the inversion of the estimated inertia matrix (Wang & Xie 2011), which is not always ensured by the usual adaptive control laws, since they do not guarantee that the estimated matrix is positive definite or even well-conditioned. To overcome that problem, it is sufficient to ensure that the estimated parameters be positive to obtain a positive definite estimated inertia matrix, because the sum of two positive definite matrices is positive definite.

This can be done by defining an appropriate convex region, whose interior defines the set of admissible positive parameters, and then using a projection algorithm to ensure that the parameters remain inside that region (Cheah et al. 2006*b*). Nevertheless, in discrete implementations of the projection algorithm the estimated parameters may escape from that region, thus Wang & Xie (2011) proposed an approach that guarantees the positiveness of the estimated parameters while retaining stability of the closed-loop system. Still, they observed that when the parameter update is too fast, the algorithm cannot project the estimated parameters into the region of admissible parameters.

1.1 Statement of Contributions

The main contribution of this paper is to use an adaptive controller, based on the ones proposed by Slotine & Li (1987) and Cheah et al. (2006*a*,*b*), in order to control a serial robot manipulator while solving the problem introduced by the ill-conditioning of the inertia matrix. In addition, an algorithm based on the one proposed by Wang & Xie (2011) is developed to ensure that the estimated parameters remain inside a suitable region, which means that the resulting robot inertia matrix is still positive definite (hence invertible). Furthermore, experimental results show that with a proper choice of initial parameters, the final inertia matrix is better conditioned than the original one.

2 Dynamic model

The dynamic model of a n-link serial manipulator is given by (Spong et al. 2006)

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}, \qquad (1)$$

where $\boldsymbol{q} = \begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix}^T$ is the vector of joints configuration, $\boldsymbol{M}(\boldsymbol{q})$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is a matrix with the Coriolis and centrifugal terms, $\boldsymbol{g}(\boldsymbol{q})$ is the gravity vector and $\boldsymbol{\tau}$ is the torque applied to the joints.

Usually the actuators' dynamics are small compared to the dynamics of the rigid multi-link robot (Shen & Featherstone 2003), hence the JSIM usually takes into consideration only the inertia of the links. However, sometimes it is useful to explicitly consider the inertia of the actuators, as they can help in improving the condition number of the resultant inertia matrix. Considering a robot whose links are connected through revolute joints, and assuming that the motion of each link is transmitted via a set of gears, its kinetic energy is the sum of the kinetic energies of the links and those of the rotors (Kelly et al. 2005, Siciliano et al. 2009); that is[1],

$$\mathcal{K}\left(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{\theta}}\right) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{\theta}}^T\boldsymbol{N}\dot{\boldsymbol{\theta}},$$

where $\boldsymbol{N} = \mathrm{diag}(\eta_1, \ldots, \eta_n)$ is a diagonal positive definite matrix, whose elements are the rotors' moments of inertia. When considering just

[1] In this model, coupling effects between rotors and links are neglected. According to Siciliano et al. (2009), some couplings in joints' dynamics may be reduced or eliminated when designing the structure in order to simplify the control problem.

the "spinning" rotor velocity, the angular velocity of the axes after the set of gears is given by $\dot{\boldsymbol{\theta}} = \begin{bmatrix} r_1\dot{q}_1 & \cdots & r_n\dot{q}_n \end{bmatrix}$, with r_i being the gear ratio of the i-th actuator. Therefore, the dynamic model that explicitly takes into consideration the actuators' inertia is given by

$$[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\,\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}, \quad (2)$$

where $\boldsymbol{M}_m = \operatorname{diag}\left(\eta_1 r_1^2, \ldots, \eta_n r_n^2\right)$.

3 Motion control laws

This section first presents some well-known controllers and discusses their behaviors with respect to stability, steady-state error, and the conditioning of the JSIM. Then it presents an adaptive controller, based on the ones proposed by Slotine & Li (1987) and Cheah et al. (2006*a*,*b*), in order to control a serial robot manipulator while solving the problem introduced by the ill-conditioning of the inertia matrix. In addition, an algorithm based on the one proposed by Wang & Xie (2011) is developed to ensure that the estimated parameters remain inside a suitable region, which means that the resulting robot inertia matrix is still positive definite (hence invertible).

3.1 Inverse dynamics with feedback linearization

A common technique to control a robot manipulator modeled by (1) is to design a control law based on inverse dynamics with feedback linearization (Spong et al. 2006); that is,

$$\boldsymbol{u} = \boldsymbol{M}(\boldsymbol{q})\,\boldsymbol{a}_q + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}), \quad (3)$$

where the control input $\boldsymbol{u} \triangleq \boldsymbol{\tau}$ is applied to low-level (joints) torque controllers and $\boldsymbol{a}_q \triangleq \ddot{\boldsymbol{q}}$ is the control law designed to stabilize the linearized closed-loop system.

One can define an additional control law as (Kelly et al. 2005)

$$\boldsymbol{a}_q = \ddot{\boldsymbol{q}}_d - \boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{K}_p\tilde{\boldsymbol{q}}, \quad (4)$$

where $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are symmetric positive definite design matrices and $\tilde{\boldsymbol{q}} \triangleq \boldsymbol{q} - \boldsymbol{q}_d$ denotes the error of the joints. This controller is asymptotically stable in the Lyapunov sense (Kelly et al. 2005).

Shen & Featherstone (2003) showed that the control law (3) behaves poorly whenever the matrix $\boldsymbol{M}(\boldsymbol{q})$ is ill-conditioned. Due to the difference in the singular values of $\boldsymbol{M}(\boldsymbol{q})$, the torque of each joint calculated from the inverse dynamics control law (3) can be very different, even if the joints accelerations are the same. This way, if the inertia along a specific joint is very small, no matter how large the position/velocity error or PD-coefficient is, the correction torque applied on that joint will be still small compared to the dominant torque, which may result in some undesired stationary error in that joint.

3.2 PD controller

To circumvent the ill-conditioning of the JSIM, Shen & Featherstone (2003) proposed the control law

$$\boldsymbol{u} = -\boldsymbol{K}_p\tilde{\boldsymbol{q}} - \boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}), \quad (5)$$

which yields an asymptotically stable closed-loop system if the PD gains are properly chosen (Shen & Featherstone 2003, Kelly et al. 2005).

The PD controller (5) directly converts the joint position/velocity error to drive torque and is not affected by the ill-conditioning of the JSIM. Yet, the inverse dynamics controller (3)-(4) should achieve better accuracy since it has complete knowledge about the robot dynamics (Shen & Featherstone 2003). Still, the control law (5) only considers the regulation problem, and does not work for a trajectory tracking problem.

3.3 Adaptive control

Considering the necessity of guaranteeing the JSIM's positiveness and the assumption of previous knowledge of the robot's kinematics and dynamics, an adequate alternative is the adaptive controller proposed by Cheah et al. (2006*a*) for the purpose of finding a suitable matrix $\boldsymbol{M}_m$ in (2). The adaptive controller is composed of two steps: i) the control law and ii) the adaptation law.

Regarding the adaptation law, a sliding vector is defined to restrict the error to a sliding surface, which is required to eliminate the steady-state position error (Slotine & Li 1987). In the joint space, the adaptive sliding vector is defined as

$$\boldsymbol{s} \triangleq \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_r, \quad (6)$$

where $\dot{\boldsymbol{q}}_r = \dot{\boldsymbol{q}}_d - \alpha(\boldsymbol{q} - \boldsymbol{q}_d)$, with α being a positive constant and $\boldsymbol{q}_d$ is the vector of desired joints configurations.

Substituting (6) and its derivative in (2) yields

$$\begin{aligned}&[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\,\dot{\boldsymbol{s}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\boldsymbol{s} + \boldsymbol{g}(\boldsymbol{q}) \\ &\quad + [\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\,\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}}_r = \boldsymbol{\tau}. \quad (7)\end{aligned}$$

Since we assume that the dynamic parameters of the links are known with sufficient accuracy, only the matrix $\boldsymbol{M}_m$ (which is related to the joints' inertia) needs to be estimated. In addition, as the robot dynamic model is linear in a set of physical parameters and its linear combinations (Cheah et al. 2006*a*), it is possible to rewrite the last terms of (7) as

$$\begin{aligned}&[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\,\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}}_r + \boldsymbol{g}(\boldsymbol{q}) = \\ &\qquad \boldsymbol{Y}_m(\ddot{\boldsymbol{q}}_r)\,\boldsymbol{a}_m + \boldsymbol{v}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r), \quad (8)\end{aligned}$$

where $\boldsymbol{v}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r) \in \mathbb{R}^n$ is a vector containing the known dynamic model (i.e.,

$\boldsymbol{v}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r) = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_r + \boldsymbol{g}(\boldsymbol{q})$), $\boldsymbol{Y}_m(\ddot{\boldsymbol{q}}_r) = \mathrm{diag}(\ddot{\boldsymbol{q}}_r)$ is the regressor, and $\boldsymbol{a}_m = \begin{bmatrix}\eta_1 r_1^2 & \cdots & \eta_n r_n^2\end{bmatrix}^T$ is the (constant) parameter vector.

Therefore, the adaptive tracking control presented by Cheah et al. (2006*a*), modified to be in the joint space, is given by

$$\boldsymbol{u} = -\boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{K}_p\tilde{\boldsymbol{q}} + \boldsymbol{v}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r) + \boldsymbol{Y}_m(\ddot{\boldsymbol{q}}_r)\hat{\boldsymbol{a}}_m \tag{9}$$

and the actuator adaptation law is

$$\dot{\hat{\boldsymbol{a}}}_m = -\boldsymbol{L}_m\boldsymbol{Y}_m^T(\ddot{\boldsymbol{q}}_r)\boldsymbol{s}, \tag{10}$$

where $\mathrm{diag}(\gamma_1,\ldots\gamma_n) = \boldsymbol{L}_m \in \mathbb{R}^{n\times n}$ is a diagonal positive-definite matrix that determines the convergence rate of the adaptive parameters.[2]

Theorem 1 *The closed-loop system given by (2) under control law defined by (9) and (10) is asymptotically stable.*

Proof: Assuming $\boldsymbol{u} \triangleq \boldsymbol{\tau}$, the closed loop dynamics is obtained by combining (7) and (8) and making it equal to (9), which results in

$$[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\dot{\boldsymbol{s}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{s} + \boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_p\tilde{\boldsymbol{q}} + \boldsymbol{Y}_m(\ddot{\boldsymbol{q}}_r)\Delta\boldsymbol{a}_m = 0, \tag{11}$$

where $\Delta\boldsymbol{a}_m = \boldsymbol{a}_m - \hat{\boldsymbol{a}}_m$.

By choosing the Lyapunov candidate function $V \triangleq V(\boldsymbol{s},\boldsymbol{\Delta a}_m,\tilde{\boldsymbol{q}})$ as

$$V \triangleq \frac{1}{2}\boldsymbol{s}^T[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\boldsymbol{s} + \frac{1}{2}\Delta\boldsymbol{a}_m^T\boldsymbol{L}_m^{-1}\Delta\boldsymbol{a}_m + \frac{1}{2}\tilde{\boldsymbol{q}}^T(\boldsymbol{K}_p + \alpha\boldsymbol{K}_v)\tilde{\boldsymbol{q}},$$

its derivative is

$$\dot{V} = \boldsymbol{s}^T[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\dot{\boldsymbol{s}} + \frac{1}{2}\boldsymbol{s}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{s} - \Delta\boldsymbol{a}_m^T\boldsymbol{L}_m^{-1}\dot{\hat{\boldsymbol{a}}}_m + \tilde{\boldsymbol{q}}^T(\boldsymbol{K}_p + \alpha\boldsymbol{K}_v)\dot{\tilde{\boldsymbol{q}}}. \tag{12}$$

Substituting $[\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m]\dot{\boldsymbol{s}}$ and $\dot{\hat{\boldsymbol{a}}}_m$ from (11) and (10), respectively, in (12) yields

$$\dot{V} = \boldsymbol{s}^T\left[-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{s} - \boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{K}_p\tilde{\boldsymbol{q}}\right] + \frac{1}{2}\boldsymbol{s}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{s} + \tilde{\boldsymbol{q}}^T(\boldsymbol{K}_p + \alpha\boldsymbol{K}_v)\dot{\tilde{\boldsymbol{q}}}. \tag{13}$$

Since the matrix $\boldsymbol{A} \triangleq \dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is skew-symmetric (Spong et al. 2006) and $\boldsymbol{s}^T\boldsymbol{A}\boldsymbol{s} = 0$, $\forall\boldsymbol{s}\in\mathbb{R}^n$ (Cheah et al. 2006*a*), then

$$\dot{V} = -\boldsymbol{s}^T\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{s} - \boldsymbol{s}^T\left(\boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_p\tilde{\boldsymbol{q}}\right) + \frac{1}{2}\boldsymbol{s}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{s} + \tilde{\boldsymbol{q}}^T(\boldsymbol{K}_p + \alpha\boldsymbol{K}_v)\dot{\tilde{\boldsymbol{q}}}.$$

Using (6), we obtain

$$\dot{V} = -\dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{K}_v\dot{\tilde{\boldsymbol{q}}} - \tilde{\boldsymbol{q}}^T\alpha\boldsymbol{K}_p\tilde{\boldsymbol{q}}. \tag{14}$$

Therefore, $\dot{V} \le 0$, which indicates that the closed loop system is stable. From (14), we conclude that $\dot{V} = 0$ if and only if $\tilde{\boldsymbol{q}}$ and $\dot{\tilde{\boldsymbol{q}}}$ are zero, which means that the system stabilizes *asymptotically* at the equilibrium point $\left(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}\right) = (\boldsymbol{0},\boldsymbol{0})$, as desired. □

Remark 2 *Since the function V depends on $\boldsymbol{s}$, $\boldsymbol{\Delta a}_m$, and $\tilde{\boldsymbol{q}}$, then at the equilibrium point $\boldsymbol{s} = \dot{\tilde{\boldsymbol{q}}} + \alpha\tilde{\boldsymbol{q}} = \boldsymbol{0}$, but there is no guarantee that $\boldsymbol{\Delta a}_m$ will converge to zero as well (if $\dot{V} = 0$ and $\boldsymbol{\Delta a}_m \ne \boldsymbol{0}$, then $V > 0$); therefore, there is no guarantee that the estimated parameters will converge to their actual values. However, this is not necessary to ensure the asymptotic stability of $\left(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}\right) = (\boldsymbol{0},\boldsymbol{0})$.*

Although controller (9) ensures asymptotic stability of $\left(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}\right) = (\boldsymbol{0},\boldsymbol{0})$, there is no guarantee that the estimated parameters are positive. However, in order to have a physical meaning, the inertia matrix must be positive definite. To guarantee that, the matrix $\boldsymbol{M}_m$ added to the JSIM must also be positive definite, since the sum of two positive definite matrices are also positive definite. Therefore an algorithm based on the one proposed by Wang & Xie (2011) is developed in the next section to ensure that all estimated parameters remain positive.

3.3.1 Estimation of positive parameters

In order to guarantee the positive definiteness of the estimated matrix $\boldsymbol{M}_m$ in (2), we define a convex region for the parameter space that correspond to the admissible parameter set (Wang & Xie 2011), and ensure that the estimated parameters are always projected onto this set. Since our goal is to compensate for uncertainties in the inertia of the joints, the parameter vector is given by $\boldsymbol{a}_m \triangleq \begin{bmatrix}a_1 & \cdots & a_n\end{bmatrix}^T$, where $a_i = \eta_i r_i^2$ and the convex region for each joint i is defined as

$$\Omega_i \triangleq \left\{\eta_i r_i^2 \ge \beta \,:\, \beta \in (0,\infty)\right\},$$

where β is the lower bound for all a_i. If each estimated parameter a_i is positive, then the matrix $\boldsymbol{M}_m$ in (2) is positive definite. Therefore, if $\hat{a}_i \in \Omega_i$, $\forall i$ then $\boldsymbol{M}_m > 0$.

Let us consider a function $f_i(\hat{a}_i)$ such that $f_i(\hat{a}_i) \le 0$ and

$$f_i(\hat{a}_i) = -\hat{a}_i + \beta, \tag{15}$$

where i corresponds to the i-th joint. When $f_i(\hat{a}_i) \le 0$, the estimated parameter is inside the convex region (or on its boundary), and hence positive. If $f_i(\hat{a}_i) > 0$ the parameter $\hat{a}_i$ is outside the admissible parameter set, and then it is necessary to project it onto the set Ω_i.

[2] Since $\boldsymbol{Y}_m^T(\ddot{\boldsymbol{q}}_r)$ is a diagonal matrix, it is easy to see that each element $\dot{\hat{a}}_i$ of $\dot{\hat{\boldsymbol{a}}}_m$ is given by $\dot{\hat{a}}_i = -\gamma_i\ddot{q}_{ri}s_i$, where $\ddot{\boldsymbol{q}}_r = \begin{bmatrix}\ddot{q}_{r1} & \cdots & \ddot{q}_{rn}\end{bmatrix}^T$ and $\boldsymbol{s} = \begin{bmatrix}s_1 & \cdots & s_n\end{bmatrix}^T$.

In order to ensure the estimation of positive parameters, (10) is redefined as (Wang & Xie 2011)

$$\dot{\hat{a}}_i = \begin{cases} -\gamma_i \lambda \nabla f_{i,\hat{a}_i}, & \text{if } f_i \geq 0 \text{ and } \nu_i \nabla f_{i,\hat{a}_i} \geq 0 \\ \nu_i, & \text{otherwise,} \end{cases} \tag{16}$$

where $\nu_i \triangleq -\gamma_i \ddot{q}_{ri} s_i$ is the i-th element of the *nominal* adaptation vector $\dot{\hat{\boldsymbol{a}}}_m$ in (10), γ_i is the i-th element of the diagonal of $\boldsymbol{L}_m$, $\ddot{q}_{ri}$ is the i-th element of $\ddot{\boldsymbol{q}}_r$, s_i is the i-th element of $\boldsymbol{s}$, the scalar λ is a positive value and $\nabla f_{i,\hat{a}_i} = df_i/d\hat{a}_i = -1$.

Since the discrete form of this algorithm is necessary when implementing it in a digital computer, we use the Euler integration to update the parameters $\hat{a}_i$ in (16) according to

$$\hat{\boldsymbol{a}}_m [k+1] = \hat{\boldsymbol{a}}_m [k] + \dot{\hat{\boldsymbol{a}}}_m [k] T,$$

where T is the sampling period. The value of λ is determined by the solution of the equation

$$f_i (\hat{a}_i [k+1]) = f_i (\hat{a}_i [k] - \gamma_i \lambda \nabla f_{i,\hat{a}_i} T) = 0, \tag{17}$$

and indicates the value necessary to project the parameter onto the boundary of Ω_i in one step. From (15) and (17) we obtain

$$-(\hat{a}_i [k] - \gamma_i \lambda \nabla f_{i,\hat{a}_i} T) + \beta = 0,$$

which implies

$$\lambda = \frac{-\hat{a}_i [k] + \beta}{\gamma_i T}.$$

Remark 3 *If the sampling period T is not small enough, some estimated parameters may become temporarily negative until the projection equation (16) is applied in the next step. However, in those situations the resultant matrix $(\boldsymbol{M} + \boldsymbol{M}_m)$ can be temporarily not positive definite, which can be overcome by increasing the value of β or decreasing the sampling period.*

4 Simulation results and discussions

In order to evaluate the proposed technique, we run three simulations of a seven-link KUKA LBR4+ robot in MATLAB[3] using the DQ Robotics library:[4]

1. In the first simulation we compare the behavior of the Adaptive Controller (9)-(10) to the Adaptive Controller with Positive Parameters (ACPP) (9)-(16) with respect to the values of the estimated parameters;
2. In the second simulation we evaluate the effect of the sampling time on the parameter estimation in ACPP;
3. Last, the third simulation was performed in order to compare the ACPP to both PD and Inverse Dynamics with Feedback Linearization (IDFL).

For the sake of simulation, we considered two models. The first one, given by (1), does not consider the actuators' model and was used as the nominal model. The second one, given by (2), explicitly takes into account the actuators' model and was used as the "real" robot.

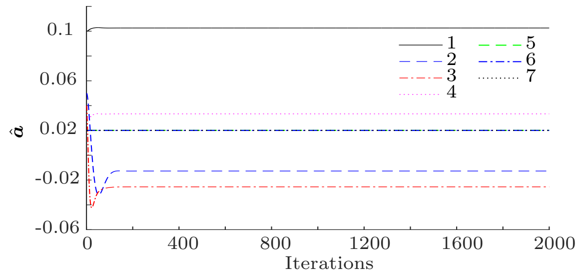
The simulation sample time was 25 ms for simulations 1 and 3, whereas simulation 2 used different sample times. The gain values for all control laws were $\boldsymbol{K}_p = 9\boldsymbol{I}$, $\boldsymbol{K}_v = 6\boldsymbol{I}$, where $\boldsymbol{I} \in \mathbb{R}^{7\times7}$ is the identity matrix. For the adaptive controller, $\alpha = 1.5$, $\boldsymbol{L}_m = 0.15\boldsymbol{I}$, $\beta = 0.015$, and the initial estimated parameters were $\hat{\boldsymbol{a}}_m [0] = \begin{bmatrix} 0.1 & 0.05 & 0.04 & 0.03 & 0.02 & 0.02 & 0.02 \end{bmatrix}^T$. The values of the gains as well as the lower bound β were chosen empirically. The choice of the initial values for the estimated parameters were also chosen empirically; however, we took into account the fact that the motors at the base of a serial manipulator are usually larger and heavier than the ones closer to the end-effector, and hence have a larger inertia. Therefore, the first values of $\hat{\boldsymbol{a}}_m [0]$ are larger than the last ones.

The robot's initial and desired configurations were given by $\boldsymbol{q}_0 = \begin{bmatrix} 0 & \pi/6 & 0 & -5\pi/9 & 0 & 0 & 0 \end{bmatrix}^T$ and $\boldsymbol{q}_d = \begin{bmatrix} 0 & \pi/2 & -\pi/2 & -5\pi/9 & 0 & 0 & 0 \end{bmatrix}^T$, respectively, and were used in all simulations. In addition, the simulation was executed in 2000 iterations, which was sufficient for all controllers, except for the IDFL, to achieve steady state (i.e., $\|\dot{\boldsymbol{q}}\| \leq 10^{-6}$).

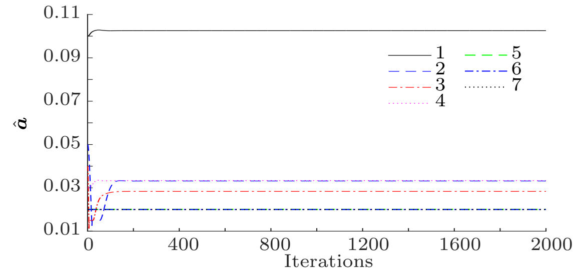
We first compare the behavior of the adaptive controller (9)-(10) to the adaptive controller with positive parameters (ACPP) (9)-(16) with respect to the values of the estimated parameters. On the one hand, Fig. 1a shows that two estimated parameters in the adaptive controller have negative values, which is undesirable because the estimated parameters should represent the joints' inertia and gear ratio; consequently they should have positive values. On the other hand, Fig. 1b shows that all estimated parameters in ACPP have positive values, as the theory predicts, which is consistent with the physical meaning of the estimated vector $\hat{\boldsymbol{a}}_m$.

A second simulation was performed to evaluate the effect of the sampling time on the parameter estimation in ACPP. Figure 2 shows that all estimated parameters are positive in steady state, although for some values of β and T it is possible for the parameters to become negative in the transient state. For instance, if $\beta = 10^{-3}$ and $\hat{\boldsymbol{a}}_{m(0)} = \begin{bmatrix} 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 \end{bmatrix}^T$, some parameters can become negative if the

[3]https://www.mathworks.com/products/matlab.html
[4]http://dqrobotics.sourceforge.net



(a) Estimated parameters *2* and *3* are negative, which is undesired.



(b) All estimated parameters are positive, as desired.

Figure 1: Estimated parameters for the adaptive control (a) and for the adaptive control with positive parameters (b).

sampling time T is not small enough. However, there is a value T for which all parameters are always positive.

Last, a third simulation was performed in order to compare the ACPP to both PD and IDFL. Figure 3 shows the error norm of all control laws. Although the error norm decreases for all controllers, the IDFL presents an oscillatory behavior, takes longer to reach the stability point and exhibits a steady-state error. The PD controller presents a smaller oscillation than the IDFL and reaches the equilibrium point faster and without steady-state error, but the oscillation at the beginning is substantial. The ACPP is smoother than both the PD and the IDFL, and the equilibrium point is reached even before the PD controller, also without steady-state error.

Figure 4 shows that, for the IDFL, the first four joints reached the desired configurations, but the fifth one took a long time to reach the desired set-point and the last two did not even reach it. This is explained by the difference in the singular values of the inertia matrix, as show in Fig. 5. Since the smallest values are almost zero, the correction torques applied to the corresponding joints are much smaller than the dominant one (i.e., the one corresponding to the largest singular value), hence stationary error is observed in those joints, as predicted by Shen & Featherstone (2003). Furthermore, the first and fourth joints have slower time response and larger overshoot for the ACPP when compared to the PD controller, although the ACPP has smoother overall error dynamics, as previously discussed, and they both achieve steady state at approximately the same time.

Figure 6 shows that the condition number of the resultant inertia matrix (i.e., $\boldsymbol{M}(q) + \boldsymbol{M}_m$) in the ACPP is much smaller than the condition number of the inertia matrix of the nominal model (i.e., $\boldsymbol{M}$), which implies that $\boldsymbol{M}(q) + \boldsymbol{M}_m$ is better conditioned than $\boldsymbol{M}$. Without considering the estimated parameters, the condition number of the JSIM is close to 8×10^3, whereas the condition number of the resultant inertia matrix is close to 100. Although we only have formal guarantee that the estimated inertia matrix is always positive definite, as shown in Section 3.3.1, Fig. 6 indicates that the ACPP is capable of improving the conditioning of the robot inertia matrix.

5 Conclusion

This paper proposes a solution to an intrinsic problem of multi-link open serial chains: the ill-conditioning of the joint-space inertia matrix (JSIM) in the Euler-Lagrange equations. This problem is observed in simulation and control of many serial robot manipulators; therefore it must be solved in order to improve both the accuracy of the controllers and closed-loop stability. Indeed, when the control input is given by a double integrator (which is very common in simulation) and the JSIM must be inverted (as in control laws based on feedback linearization), an ill-conditioned matrix may result in arbitrarily large joint velocities.

An adaptive control law in joint space has been implemented to improve the condition number of JSIM. This controller estimates a positive definite matrix related to the joints' inertia, which is then added to the nominal robot inertia matrix to obtain a better conditioned JSIM. The proposed technique was compared in simulation to a proportional-derivative (PD) controller and to an inverse-dynamics feedback-linearizing (IDFL) controller. The results showed that the adaptive controller has the smoothest time response, whereas both PD and IDFL present an oscilatory behavior. In addition, both the adaptive and the PD controllers have zero steady-state error, which is not the case for IDFL.

When forcing the estimated parameters to be positive, the condition number of the resultant matrix $(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m)$ was smaller than the original $\boldsymbol{M}(\boldsymbol{q})$ in all simulations. However, there is no guarantee that the sum of two positive definite matrices will result in a better conditioned matrix. In fact, the sum $(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m)$ is better conditioned than $\boldsymbol{M}(\boldsymbol{q})$ if and only if $\boldsymbol{M}_m$ is better conditioned than $\boldsymbol{M}(\boldsymbol{q})$. This way, it is possible to initialize the estimated parameters with appropriate values such that the initial condition number of $(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m)$ is smaller than $\boldsymbol{M}(\boldsymbol{q})$. Therefore, since the condition number of

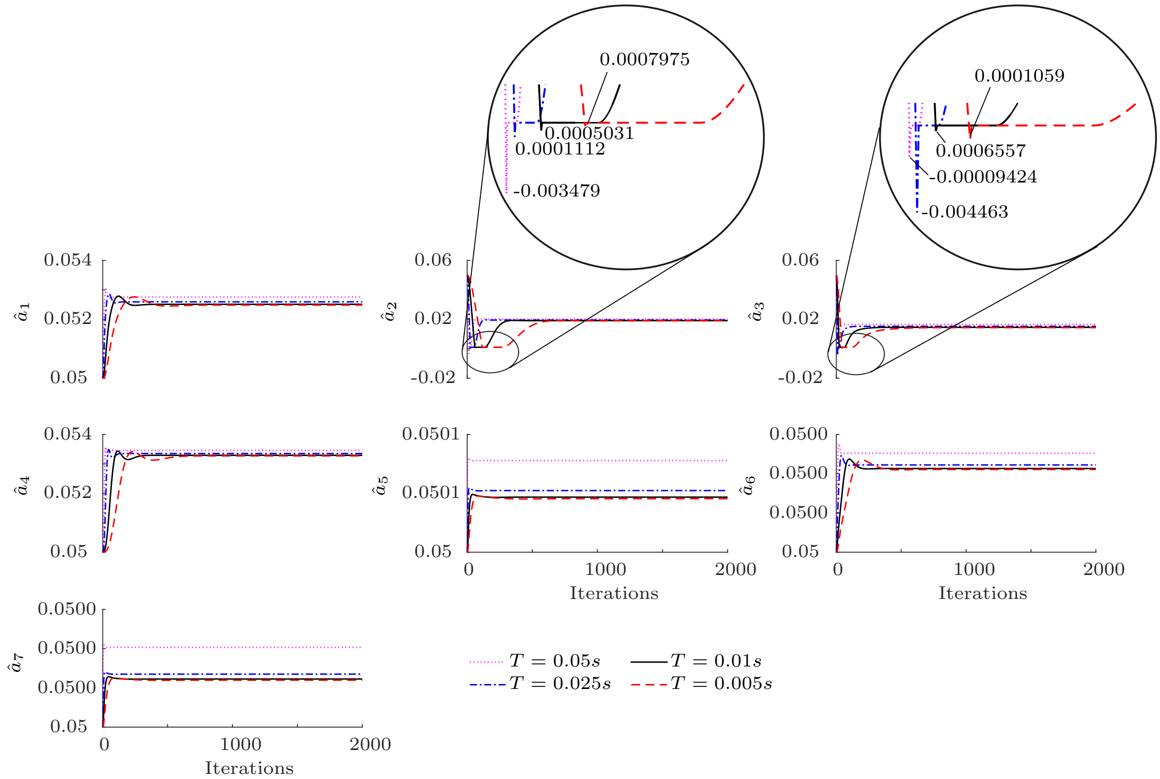


Figure 2: Estimated parameters in ACPP for four different sampling periods. For $T = 0.05\,\mathrm{s}$ there are two negative parameters, for $T = 0.025\,\mathrm{s}$ there is one negative parameter, and for $T = 0.01\,\mathrm{s}$ and $T = 0.005\,\mathrm{s}$ all parameters are positive.

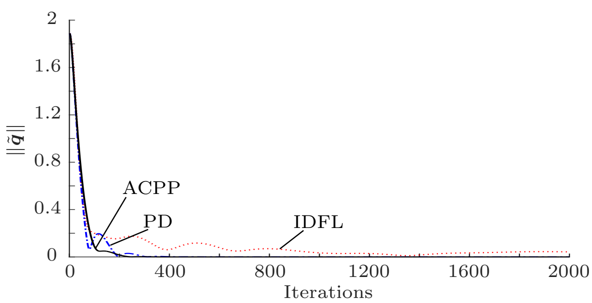


Figure 3: Norm of the vector of joints error. The adaptive controller with positive parameters (ACPP) presents the smoothest decay among all compared controllers.

$(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{M}_m)$ does not change significantly, as observed in the simulation, it remains small during the whole trajectory.

Future works will be focused on *formally* ensuring that the estimated robot inertia matrix (i.e., the one that takes into account the nominal inertia matrix plus the joints' inertia matrix) is better conditioned than the nominal inertia matrix, in addition to the implementation of the adaptive control law in task space, as well as on the stability proof of the closed-loop system that takes into account the estimation of positive parameters, and the implementation on a real robot.

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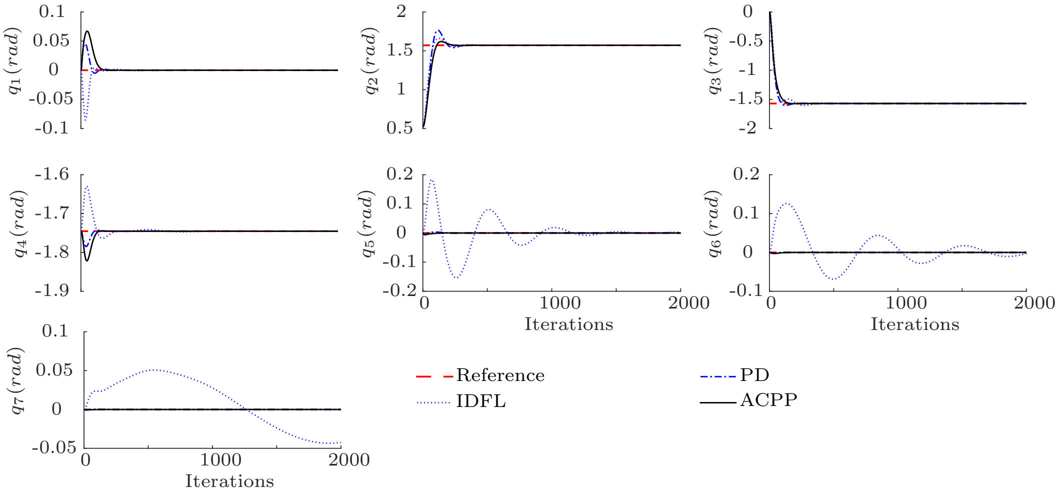

Figure 4: Joints configurations during simulation. For the IDFL, the last two joints did not reach the desired values due to the difference in the singular values of the JSIM (i.e., JSIM had a large condition number near the desired configuration).

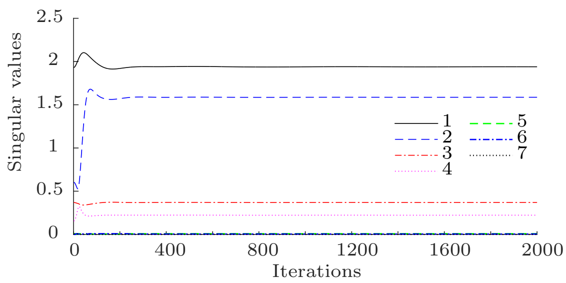

Figure 5: Singular values related to each robot's joint during the simulation of the IDFL. Since the last three singular values are much smaller than the largest one, the resultant torque related to the last three joints are almost inexpressive compared to the dominant torque.

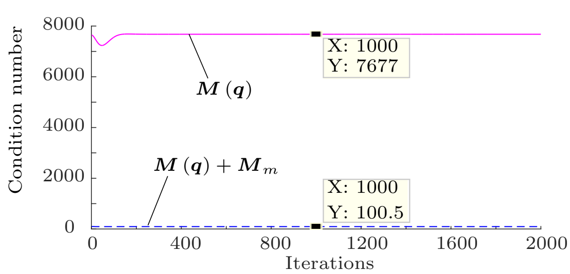

Figure 6: Condition number of the nominal JSIM ($\boldsymbol{M}$) and of the resultant JSIM ($\boldsymbol{M}\,(\boldsymbol{q}) + \boldsymbol{M}_m$).

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