

# Capacitor placement in unbalanced distribution networks to minimize reactive and unbalance power losses

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**Abstract:** In this paper, the optimized placement and sizing of capacitors for the compensation of the reactive power and the unbalance power in medium voltage distribution networks with unbalanced loads is proposed. The proposed capacitor allocation reduces power losses in power networks caused by reactive power demand and unbalance. The powers are calculated as proposed in the IEEE STD 1459-2010, and presented real values of electrical magnitudes in unbalanced three-phase systems. The results of the capacitor placement obtained from the proposed method are compared with the placement of single-phase capacitor and the placement of balanced three-phase capacitor bank. The results allow to observe the efficient performance of the proposed method for reducing power losses in a 34-bus medium voltage power network with the presence of the neutral conductor. Four cases are simulated with results optimized by the Chu-Beasley genetic algorithm. The purpose of the optimization algorithm is to minimize line loss and capacitor costs.

**Keywords:** Capacitor allocation, effective power theory, ideal compensation method, neutral power losses, unbalanced distribution network.

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## 1. INTRODUCTION

In medium voltage (MV) networks, the unbalance power of the unbalanced power loads causes power losses in the conductors (including the neutral) and voltage unbalance. These unbalances and reactive power in the three-phase network can be compensated via the capacitor placement at the three-phase buses according to the ideal compensation method. The ideal compensation method for unbalanced loads is detailed in Gyugyi, Otto, Putman, 1978, Lee, Wu, 1993.

The ideal compensation of an unbalanced three-phase power load fed by a symmetrical source can be performed by connecting capacitors and/or inductors with the load to compensate for reactive power and unbalance power. The load compensation results in a balanced system with unity power factor.

To obtain balance, this method compensates for the negative- and zero-sequence components of the electrical current of the unbalanced power loads, i.e., the unbalance power of the loads. To compensate for the reactive power, the method compensates for the imaginary part of the positive-sequence component of the current. The compensation of the negative component of the current is obtained by passive compensators connected in delta and the compensation of the zero sequence electrical current is through compensators in wye connected to the neutral.

The IEEE STD 1459-2010 (IEEE STD 1459-2010, 2010) quantifies the powers in unbalanced and symmetrical systems. The effective apparent power is decomposed into active power, reactive power, and unbalance power. The unbalance power, caused by the unbalanced components of the current, does not perform work as well as the reactive power, as explained in Emanuel, 1993.

This ideal compensation method has been approached in the literature via isolated load compensation (Bronstein, et al, 2016, Kiyani, Aydemir, 2014, Sainz, Pedra, Caro, 2005, Pati, Sahu, 2012). However, the ideal compensation method applied in MV distribution networks with the presence of the neutral conductor has not been considered in the literature. Therefore, none of the works presented in the literature consider the effective compensation of unbalances in MV distribution networks via capacitor allocation.

There are few papers on capacitor allocation in unbalanced networks. References Subrahmanyam, Radhakrishna, 2010, Kim, You, 1999, Chiang, et al, 1994, allocate three-phase banks of capacitor in unbalanced systems for the reactive power compensation, and in Pereira, Fernandes, Aoki, 2018, are allocated capacitor banks and voltage regulators in MV unbalanced networks having as one of the objectives the minimization of the voltage unbalance deviation.

The compensation of power losses by single-phase capacitor allocation was discussed in Murty, Kumar, 2013, 2014, Martins, et al, 2021, Araujo, et al, 2018, for unbalanced three-phase power systems. Although Araujo, et al, 2018, uses banks in delta and wye, the banks are balanced.

In Carpinelli, et al, 2005, Esmaeilian, Fadaeinedjad, 2013, Eajal, El-Hawary, 2010, single-phase capacitor are placed in unbalanced three-phase systems to compensate for the losses and harmonic distortions, and in Esmaeilian, Fadaeinedjad, 2013, the unbalanced voltage reduction is also included in the objective of the problem.

The ideal compensation method is applied to unbalanced systems (Semensato, 2019); however the comparison with traditional allocation methods is not described.

The unbalance compensation is proposed by Zhu, Chow, Zhang, 1998, with the phase change method. In Zeng, et al, 2019, the unbalance compensation is obtained by proposed device based on Static Var Compensators (SVCs). In Gupta, Swarnkar, Niazi, 2011, the reduction in the unbalance is obtained by modifying the transformer winding.

The proposal of this paper is the optimized placement and sizing of shunt capacitor in three-phase buses, connected in delta or wye, in MV distribution power systems with unbalanced loads and with the presence of the neutral conductor, in order to compensate for the unbalance power together with the reactive power in the network. The aim is to minimize the sum of the costs of the losses in the conductors of the power network and the costs of the capacitors. But now, unlike the previously published in MV networks, this proposal compares ideal compensation method with the placement of single-phase capacitor and the placement of balanced three-phase capacitor bank.

Neutral analysis is essential for unbalanced networks. A capacitor allocation without this analysis may not reduce the excessive power losses in the neutral. In Ochoa, et al, 2005, the IEEE 34-bus network was expanded to four-wire, representing the phases, the neutral conductor, and the ground. The IEEE 34-bus network obtained by the reduction of Kron, incorporating the effects of neutral conductor and the ground in the phases, presents power losses unlike the original system that is not reduced. This shows the importance of the representation of the neutral conductor and the ground in unbalanced networks.

The optimization of the solution is obtained by metaheuristic applied to the problem of the capacitor placement. The electrical magnitudes are calculated by the IEEE STD 1459-2010, which defines the calculations for unbalanced three-phase power systems (effective power theory). The effective power theory is applied for measurement in unbalanced networks, such as effective power factor and unbalanced power.

## 2. EFFECTIVE POWER THEORY

Although the physical concept of active and reactive power is well defined (active power performs work but reactive does not), depending on the calculation used for these powers, the values do not represent the physical concept in unbalanced three-phase systems. The IEEE STD 1459-2010 defines three apparent powers for unbalanced three-phase systems: arithmetic apparent power, vector apparent power, and effective apparent power. The three apparent powers have equal values for the application in balanced and symmetrical three-phase systems; however, in unbalanced systems they have different values. In Czarnecki, Haley, 2015, it is stated that the power factor in unbalanced three-phase systems only shows the true value if it is calculated by the effective apparent power.

The effective power theory is the result of the work of Emanuel, 2011, with the objective of measuring the efficiency in the use of power lines and electrical equipment. The effective apparent power ( $S_e$ ) in unbalanced and symmetrical three-phase systems, as shown in Fig. 1, is

decomposed into positive-sequence active power ( $P^+$ ), positive-sequence reactive power ( $Q^+$ ), and unbalance power ( $S_D$ ). The effective apparent power supplied by the sinusoidal and symmetric voltage source is given by (1).

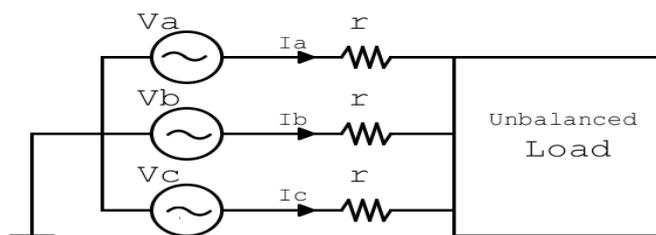


Fig. 1. Unbalanced and symmetrical three-phase system.

$$S_e^2 = P^{+2} + Q^{+2} + S_D^2 \quad (1)$$

The positive-sequence active and reactive powers are obtained from the positive-sequence component of the electrical current ( $I^+$ ), according to (2) and (3). The unbalance power is obtained from the negative-sequence components ( $I^-$ ) and zero-sequence ( $I^0$ ) of the electrical current, according to (4), (5), and (6).

$$P^+ = 3VI^+ \cos(\theta^+) \quad (2)$$

$$Q^+ = 3VI^+ \sin(\theta^+) \quad (3)$$

$$Q^- = 3VI^- \quad (4)$$

$$Q^0 = 3VI^0 \quad (5)$$

$$S_D = \sqrt{Q^{-2} + Q^{02}} = 3V\sqrt{I^{-2} + I^{02}} \quad (6)$$

Here, ( $V$ ) represents the magnitude of the single-phase voltages of the symmetric source ( $V_a, V_b, V_c$ ), and ( $\theta^+$ ) is the angular difference between the source voltage and the positive-sequence component of the electrical current. The negative-sequence unbalance power and zero-sequence are represented by ( $Q^-$ ) and ( $Q^0$ ), respectively.

The reactive power and unbalance power do not perform work. Although the instant unbalance power of the negative-sequence and zero-sequence calculated by phase has a non-zero mean value, this value (energy) is changed between the phases of the system. The unbalance power is null in unbalanced systems when calculated by the vector power, i.e., through traditional calculation (Semensato, 2020).

The unbalance power is a measure related to the power losses in the three-phase network and can be compensated, thus increasing the efficiency of the power network.

In the unbalanced and symmetrical three-phase system, shown in Fig. 1, the effective apparent power is calculated directly by the symmetric source voltage and the effective electrical current ( $I_e$ ), according to (7).

In the unbalanced system, the effective electrical current shows the same power losses in the electrical lines as the unbalanced currents ( $I_a, I_b, \text{ and } I_c$ ), according to (8) and (9) (Emanuel, 1993). And  $r$  is the electrical resistance of the line, in ohms.

$$S_e = 3V\sqrt{I^{+2} + I^{-2} + I^{02}} = 3VI_e \quad (7)$$

$$3rI_e^2 = r(I_a^2 + I_b^2 + I_c^2) \quad (8)$$

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} \quad (9)$$

The effective power factor is obtained by (10).

$$fp_e = \frac{P^+}{S_e} \quad (10)$$

The effective power factor presents the real efficiency in the use of the electrical power network. Therefore, for effective compensation of the power factor, it is necessary to compensate the reactive and unbalance power.

### 3. PROBLEM FORMULATION

The problem is to minimize the objective function and meet the operational constraints.

#### 3.1. Objective function

The objective function ( $OF$ ) consists of two cost functions. The objective function is formed by the costs of power losses in the distribution lines and the costs of the capacitors. The simulation period is 20 years corresponding to the lifetime of the capacitors (Szuovivivski, Fernandes, Aoki, 2012). The price of the electrical energy, obtained from CCEE, 2022, and the load curve are constants in this period. The  $OF$  is given by (11).

$$OF = \left( \sum_{t=0}^{ys-1} \frac{C_E(Lf + KLn)}{(1+d)^t} \right) + C_f Capf + C_l Capl \quad (11)$$

$C_E$  = Cost of electrical energy (42 US\$/MWh).

$ys$  = Time period equal to lifetime of the capacitors (20 years).

$Lf$  = Active power losses in the phases in the period of one year or 8760 hours (MWh).

$Ln$  = Active power losses in the neutral conductor and ground in the period of one year or 8760 hours (MWh).

$d$  = Annual interest rate (6.82 %).

$C_f$  = Cost of capacitor connected between phase and neutral (5.1 US\$/kVAr).

$C_l$  = Cost of capacitor connected between phases (3.1 US\$/kVAr).

$Capf$  = Sum of the rated power of the capacitors connected between phase and neutral, in kVAr.

$Capl$  = Sum of the rated power of the capacitors connected between phases, in kVAr.

$K$  = Penalty in the cost of power losses in the neutral and ground conductor.

The capacitors connected between phases are cheaper than those connected between the phase and neutral of same power.

In the power losses of the neutral conductor and ground, a penalty is applied, increasing the costs by  $K$  times. This penalty considers a greater importance in reducing the power losses of neutral and ground.

In the substation, the unbalance energy due to the zero- and negative-sequence components of the electrical current is undesirable. Unbalance energy ( $E_D$ ), in MVAh, is equal to the value of unbalance power in the substation multiplied by one year ( $S_D 8760$ ).

The costs of active power losses after the first year are referred to the present by the interest rate.

#### 3.2. Electrical voltage level constraint

The electrical voltage levels in the network, measured between the phase and neutral, must be in the range indicated in (12) (ANEEL, 2022), in  $p.u.$

$$0.93 \leq V_{pn} \leq 1.05 \quad (12)$$

#### 3.3. Unbalance factor constraint

The unbalance factor ( $UF$ ) measures the relationship between the negative-sequence voltage ( $V^-$ ) and the positive-sequence voltage ( $V^+$ ), according to (13) (ANEEL, 2022). It is a calculated index for a three-phase bus.

$$UF = \frac{V^-}{V^+} 100 \leq 2\% \quad (13)$$

#### 3.4. Power factor constraint

The effective power factor of the unbalanced network is calculated at the energy substation by (10). Its constraint is indicated in (14), and it is considered the reference value in MV networks (ANEEL, 2022).

$$fp_e \geq 0.92 \quad (14)$$

The power factor is a parameter of energy savings in electrical power lines. The lower the effective powers factor in the substation, the greater is the amount of non-active power in the transmission lines. The non-active power measured at the substation refers to the reactive power and the unbalance power in (3) and (6).

#### 4. OPTIMIZATION ALGORITHM

The optimization algorithm adopted is a metaheuristic, specialized in the search for the best solution among a space of solutions to minimize the objective function in (11). The metaheuristic used is the Chu-Beasley genetic algorithm (CBGA) (Guimarães, Castro, 2011).

The individual in the CBGA is a vector that represents the solution of the problem. Each gene or column of this vector corresponds to a magnitude of the simulation. The genes presented in Fig. 2 represent the capacitor allocation in a bus. The individual, in this paper, is formed by the placement bus, the type of connection or phase of the capacitor, and the values of the rated powers of the capacitors.

The individual for the proposed method, based on the ideal compensation method, Fig. 2a, shows in the first column the number of the placement bus of the capacitor, the second column the type of capacitor connection, "1" being the wye connection (wye-center is connected to the neutral conductor) and "2" the delta connection, and the third, fourth, and fifth columns represent the rated values of the capacitors, in kVAR, in the phases *a*, *b*, and *c*, respectively, when the connection is wye. If the connection is delta the capacitors are between phases *a-b*, *b-c*, and *c-a*, respectively, and so on for the other allocation buses. However, if the allocation bus is not three-phase, phase-to-neutral capacitor is allocated to the existing phases.

The individual for the method of the single-phase capacitor allocation, Fig. 2b, shows in the first column the number of the placement bus, the second column the electrical phase of the placement capacitor, where "1" is phase *a*, "2" is phase *b*, and "3" is phase *c*, the third column is the rated value of the capacitor, in kVAR, connected between the phase and neutral, and so on for the other allocation buses.

The individual for the allocation method of three-phase capacitor bank, Fig. 2c, shows in the first column the number of the placement three-phase bus of the bank, the second column the type of capacitors connection, "1" is wye and "2" is delta, the third column represents the rated value of one capacitor of the balanced three-phase bank, in kVAR, and so on for the other allocation buses.

The CBGA used for the problem of placement and sizing of capacitor in the distribution networks is presented in the flowchart of Fig. 3.

15	1	75	133	200
(a)				
14	1	100		
(b)				
22	2	75		
(c)				

Fig. 2. Representation of individuals  
 (a) Proposed method, (b) Phase method, (c) Bank method.

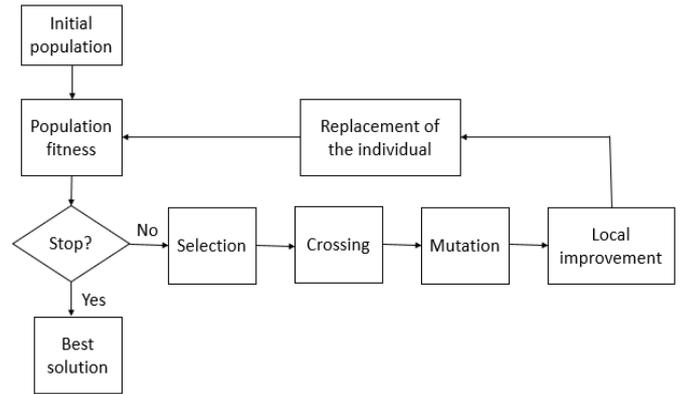


Fig. 3. Flowchart of the optimization algorithm.

CBGA uses selection, crossover, mutation and local improvement operators. The local improvement is applied only to the rated values of the capacitors. The algorithm has no mutation and crossover probability.

#### 5. CASE TEST

The radial distribution network tested is the expanded IEEE 34-bus for four-wire operation, considering the neutral and the ground as the return conductors. The representation (ANNEX) and data of the expanded IEEE 34-bus network is obtained Ciric, Feltrin, Ochoa, 2003. The pre-installed capacitors, power transformer, and voltage regulators are removed from the power network to perform the tests. The neutral conductor when grounded has a ground resistance of 5 Ω (Watson, Watson, Lestas, 2018), with the substation solidly grounded.

The shunt capacitors are modeled as constant admittance and the power loads are modeled as constant power. The three-phase loads are connected in wye (center-wye connected to the neutral), and single-phase loads are connected between phase and neutral.

The power network voltage is 25 kV (phase to ground). The bases used are 25 kV and 1 MVA (values per phase). The voltage at the substation, zero bus, is symmetrical with the value 1 p.u.

Three different ways of installing capacitor are considered, and will be referred to here as methods, as follows:

- **Proposed:** Allocation of capacitor in three-phase bus connected in delta or wye (wye-center connected to the neutral). Capacitors connected to the three-phase bus may have different values. It is based on the ideal compensation method described in Gyugyi, Otto, Putman, 1978, Lee, Wu, 1993. Single-phase capacitor can be installed in bus that are not three-phase.
- **Phase:** Allocation of single-phase capacitor in any bus and phase of the network.
- **Bank:** Allocation of capacitor bank in three-phase bus, connected in delta or wye, and the bank is balanced.

The results of the quantities in the power network are obtained before the capacitor allocation (Before) and

compared with the results of the three allocation methods described.

Four cases are simulated:

- **Case 1:** Neutral is grounded on buses 3, 8, 10, 12, 16, 19, 21, 25, 27, and 30. Where  $K = 1$ .
- **Case 2:** The neutral is isolated. The power loads in phases  $a$  and  $c$  are increased and decreased by 20%, respectively.  $K = 1$ .
- **Case 3:** Similar to case 2, but with  $K = 40$ .
- **Case 4:** Similar to case 2, but the objective function is added by the cost of the unbalance energy in the substation, as described in (15).

$$OF_4 = \left( \sum_{t=0}^{ys-1} \frac{C_E(Lf + Ln + E_D)}{(1+d)^t} \right) + C_f Capf + C_l Capl \quad (15)$$

## 6. SIMULATION RESULTS

The results for the four cases test are presented. The power flow is calculated by the backward-forward method described in Ciric, Feltrin, Ochoa, 2003, and simulated in MatLab software. The CBGA was simulated in MatLab software, with an initial population of 100 individuals, with a limit of 4,000 iterations, mutation in two points, and for the tournament operation four individuals are selected. The simulation was performed in a computer with Intel Core i7 processor, 8 GB RAM, O.S Windows 10: 64 bits. The simulation time for the tested cases is between 5 and 6 minutes. The proposal of this paper is not to evaluate the performance of the optimization algorithm in relation to another algorithm, but to compare the capacitor allocation methods. The algorithm is exhaustively tested in search of the best solutions for each case.

The quantities  $E_D$ ,  $Ln$ , and  $Ln + Lf$  in tables 1, 2, 3, and 4 are multiplied by the 20-year time period and the costs are obtained by the objective function.

The voltage profile is shown in the most loaded phase (phase  $a$ ) and neutral conductor for tested methods before and after the capacitor allocation.

The four tested cases, before allocation, have a voltage unbalance factor lower than 1.05% (small voltage asymmetry), although it presents significant values for the current unbalance factor, reaching the value of 33% (large current asymmetry). Therefore, even with the low  $UF$  value, the unbalance power in the power network is significant.  $UF$  in tables 1, 2, 3, and 4 indicates the largest unbalance factor value in the power network.

The percentage reduction pointed out in Tables 1, 2, 3, and 4 after the capacitor allocation by the tested methods is calculated as in (16).

$$Reduction (\%) = \left( 1 - \frac{Tested Method}{Before} \right) 100 \quad (16)$$

The rated values, in kVAr, of the fixed capacitors (compensators) used in the allocation are 50, 75, 100, 133, 150, 167, and 200. The commercial values of the shunt

compensators correspond to both the phase-to-neutral and the phase-to-phase connection. Capacitor with phase-to-neutral connection, the rated voltage is equal to the network phase voltage. Capacitor with phase-to-phase connection, the rated voltage is equal to the network line voltage.

### 6.1. Case 1

Table 1 is showed the results before and after the allocation of capacitors by the three methods.

Table 1. Results for case 1.

	Before	Proposed	Phase	Bank
Costs (US\$)	518,870	372,920	374,450	374,820
Reduction (%)	-	28.13	27.83	27.76
$20E_D$ (MVAh)	9919.1	14008	15554	9986.8
Reduction (%)	-	-41.22	-56.81	-0.68
$20Ln$ (MWh)	195.46	73.52	88.05	192.33
Reduction (%)	-	62.39	54.95	1.60
$20(Ln + Lf)$ (MWh)	21529	15271	15293	15404
Reduction (%)	-	29.07	28.97	28.45
$UF$ (%)	0.37	0.27	0.30	0.31
$fpe$	0.8630	0.9990	0.9986	0.9993
$Capf + Capl$ (kVAr)	-	1175	1150	1149

### 6.2. Case 2

The voltages in phase  $a$  for the two methods before and after the capacitor allocation are shown in Fig. 4, and the neutral conductor voltages for the two methods before and after the capacitor allocation are shown in Fig. 5. Table 2 shows the results before and after the allocation of capacitors by the two feasible methods. The Bank method is not feasible for this case, as the operational constraints are not met.

The convergence of the optimization algorithm (CBGA) is shown in Fig. 6 and Fig. 7 for the proposed method and the phase method, respectively, in this case.

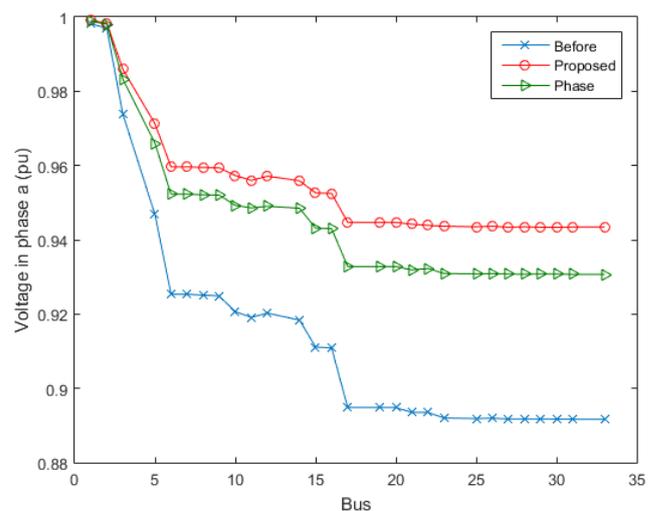


Fig. 4. Voltage in phase a (case 2).

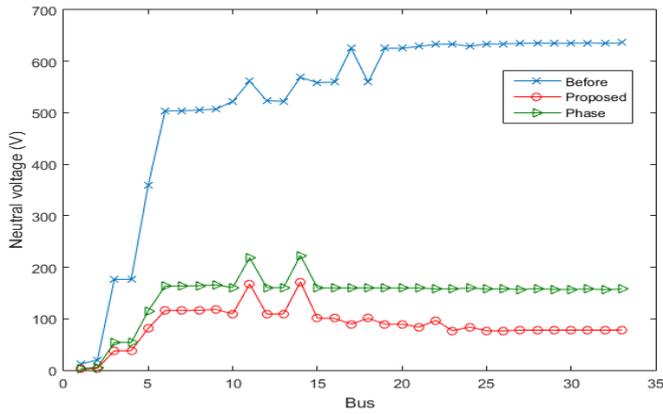


Fig. 5. Neutral voltage (case 2).

Table 2. Results for case 2.

	Before	Proposed	Phase
Costs (US\$)	569,860	375,450	396,410
Reduction (%)	-	34.12	30.44
$20E_D$ (MVAh)	84523	21286	80898
Reduction (%)	-	74.82	4.29
$20Ln$ (MWh)	1242.5	138.47	185.11
Reduction (%)	-	88.86	85.10
$20(Ln + Lf)$ (MWh)	23645	15389	16208
Reduction (%)	-	34.92	31.45
UF (%)	1.04	0.26	1.21
$f_{pe}$	0.8437	0.9978	0.9699
Capf + Capl (kVAr)	-	1183	1133

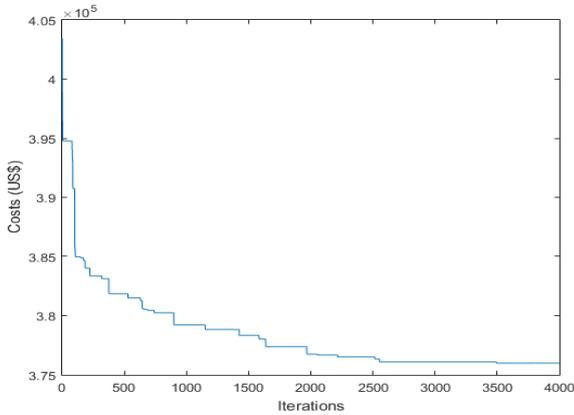


Fig. 6. Algorithm convergence for the proposed method.

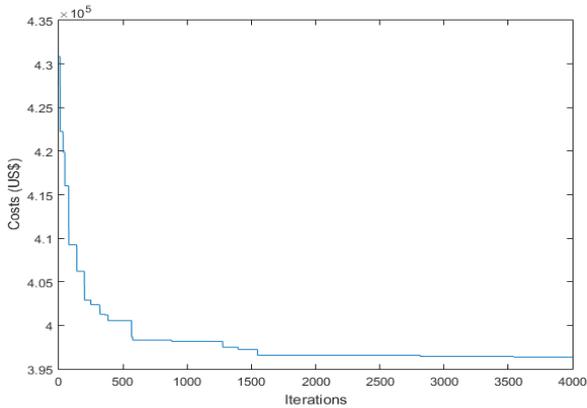


Fig. 7. Algorithm convergence for the phase method.

### 6.3. Case 3

The voltages in phase *a* for the two methods before and after the capacitor allocation are shown in Fig. 8, and the neutral conductor voltages for the two methods before and after the capacitor allocation are shown in Fig. 9. Table 3 shows the results before and after the allocation of capacitors by the two feasible methods. The Bank method is not feasible for this case.

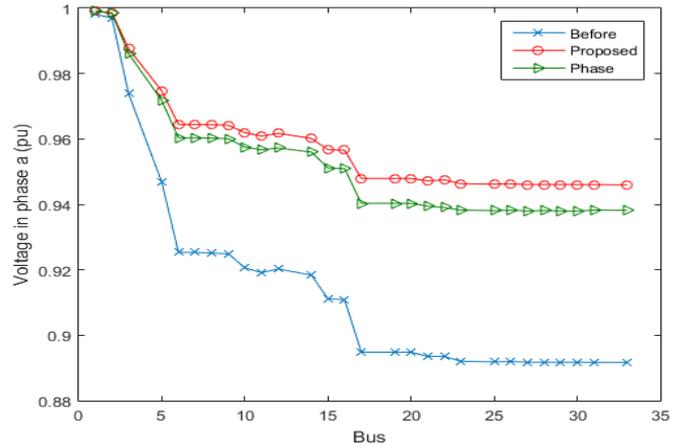


Fig. 8. Voltage in phase a (case 3).

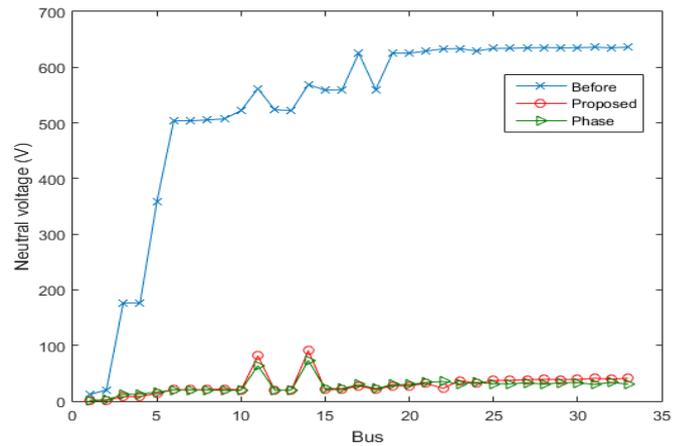


Fig. 9. Neutral voltage (case 3).

Table 3. Results for case 3.

	Before	Proposed	Phase
Costs (US\$)	1,737,800	448,290	470,970
Reduction (%)	-	74.20	72.90
$20E_D$ (MVAh)	84523	33272	96152
Reduction (%)	-	60.64	-13.76
$20Ln$ (MWh)	1242.5	74.85	73.16
Reduction (%)	-	93.98	94.11
$20(Ln + Lf)$ (MWh)	23645	15468	16428
Reduction (%)	-	34.58	30.52
UF (%)	1.04	0.42	1.37
$f_{pe}$	0.8437	0.9938	0.9590
Capf + Capl (kVAr)	-	1283	1233

### 6.4. Case 4

The voltages in phase *a* for the two methods before and after the capacitor allocation are shown in Fig. 10, and the neutral

conductor voltages for the two methods before and after the capacitor allocation are shown in Fig. 11. Table 4 shows the results before and after the allocation of capacitors by the two feasible methods.

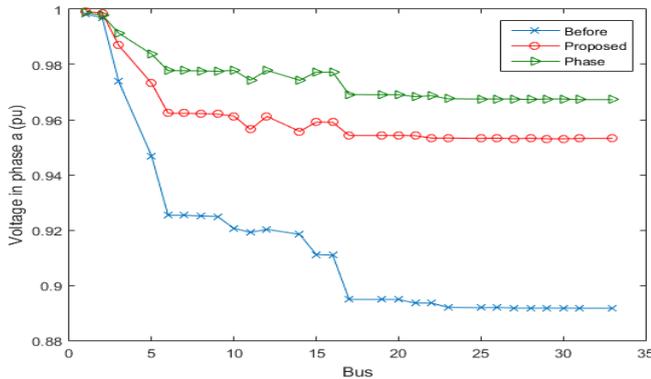


Fig. 10. Voltage in phase a (case 4).

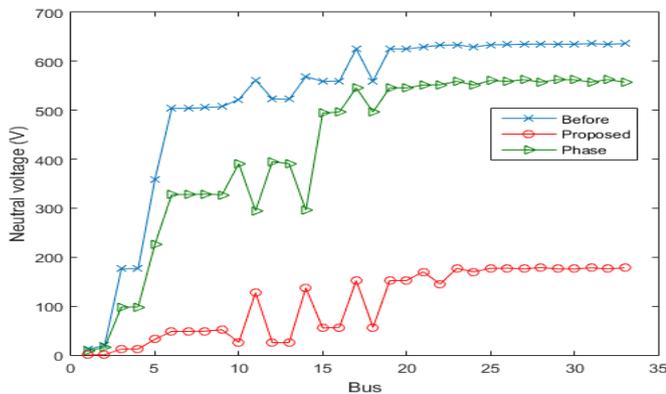


Fig. 11. Neutral voltage (case 4).

Table 4. Results for case 4.

	Before	Proposed	Phase
Costs (US\$)	2,606,900	414,560	1,872,100
Reduction (%)	-	84.10	28.19
$20E_D$ (MVAh)	84523	966.16	57543
Reduction (%)	-	98.86	31.92
$20Ln$ (MWh)	1242.5	369.62	1318
Reduction (%)	-	70.25	-6.08
$20(Ln + Lf)$ (MWh)	23645	15984	19756
Reduction (%)	-	32.4	16.45
UF (%)	1.04	0.56	1.97
$fp_e$	0.8437	0.9791	0.9368
Capf + Capl (kVAr)	-	1584	1801

The largest costs reduction (Tables 1, 2, 3, and 4) is obtained by the method proposed in this paper, proving the effectiveness of the method for the unbalance and reactive power compensation in an unbalanced power network. The other methods do not effectively compensate the unbalance power. The substation unbalance energy ( $E_D$ ) had significant reduction, more than 60% for the most unbalanced cases, only in the proposed method, in accordance with tables 2, 3, and 4. In case 4, where the unbalance energy is penalized in the objective function, the reduction is 98.86%.

Table 5. Allocated capacitors.

CASE 1	
Proposed	7a-n(50), 7c-n(75); 14a-n(75); 22a-b(150), 22b-c(75), 22c-a(133); 23a-n(100), 23b-n(167), 23c-n(150); 33a-b(50), 33b-c(100), 33c-a(50).
Phase	6a-n(50), 6c-n(75); 11a-n(75); 17a-n(75); 22b-n(100), 22c-n(100); 25a-n(75), 25b-n(75); 26c-n(100); 28a-n(133), 28c-n(50); 31b-n(167), 31c-n(75).
Bank	6a-b(50), 6b-c(50), 6c-a(50); 22a-b(100), 22b-c(100), 22c-a(100); 23a-b(50), 23b-c(50), 23c-a(50); 25a-b(50), 25b-c(50), 25c-a(50); 28a-b(133), 28b-c(133), 28c-a(133).
CASE 2	
Proposed	8a-n(50), 8c-n(75); 14a-n(100); 22a-b(200), 22b-c(133); 26a-n(150), 26c-n(75); 28a-b(200), 28b-c(200).
Phase	7c-n(75); 9a-n(50); 14a-n(100); 21b-n(100), 21c-n(150); 22a-n(150); 28a-n(200), 28b-n(133), 28c-n(50+75); 30a-n(50).
CASE 3	
Proposed	8a-n(133), 8c-n(167), 11a-n(100); 22a-n(133), 22c-n(50); 23a-b(200+167), 23b-c(200+133).
Phase	8a-n(100); 8c-n(200); 10a-n(50); 11a-n(100); 17a-n(100); 21c-n(133); 23b-n(100); 28a-n(100); 31a-n(150); 31c-n(100); 33b-n(100).
CASE 4	
Proposed	1a-b(200), 1b-c(150); 15a-b(167), 15b-c(200); 21a-n(200), 21c-n(150); 22a-b(150), 22b-c(150); 28a-n(167), 28c-n(50).
Phase	2b-n(200+167+150); 10c-n(133); 14a-n(100); 15a-n(167); 16c-n(200+200); 22a-n(167); 26a-n(167), 26b-n(50); 27a-n(100).

The proposed method reduces the unbalance factor in all tested cases (Tables 1, 2, 3, and 4).

The allocation method of three-phase capacitor bank is not useful in the unbalance compensation.

The analysis of the neutral conductor for unbalanced systems is crucial to verify the reduction of the neutral losses by the applied method. The Proposed method considerably reduced the neutral losses ( $Ln$ ), pointing to values greater than 60%, in accordance with tables 1, 2, 3, and 4. The reduction in neutral losses is greater than 90% in case 3 due to the penalty applied (Table 3).

The power factor measurement can lead to errors if it is not calculated by the effective power theory. The effective power factor decreases with the greatest load unbalance (Table 1 and 2). In case 2, before the allocation of capacitors, the effective power factor is 0.8437 and the vector power factor (traditional calculation) is 0.8638.

The bus, phases and size of the allocated capacitors in the power network for each method tested are shown in Table 5. Each element corresponds to the bus, phase(s) of connection and within the parentheses the capacitor value in kVAr. The index  $n$  represents the connection of the neutral conductor.

## 7. CONCLUSION

Herein, an effective method for the unbalance power compensation and reactive power compensation for power distribution network with unbalanced loads was proposed. Contrary to the methods described in the literature, the proposed approach considers the unbalance compensation in the capacitor allocation using the ideal compensation method. The unbalance compensation is the primary cause of the cost reduction for capacitor allocation by the proposed method in comparison to the other methods tested.

The application of the Proposed method showed that the measures of effective power factor and unbalance power are fundamental to the analysis of unbalanced distribution systems. It is necessary to compensate the unbalance power for an effective compensation of the power factor in the network. The proposed method proved to be efficient for the compensation of power losses in neutral, avoiding overload in conductor or severe voltage drops. This study may help the distribution companies in the most efficient manner for the capacitor allocation in MV networks with unbalanced loads.

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## ANNEX

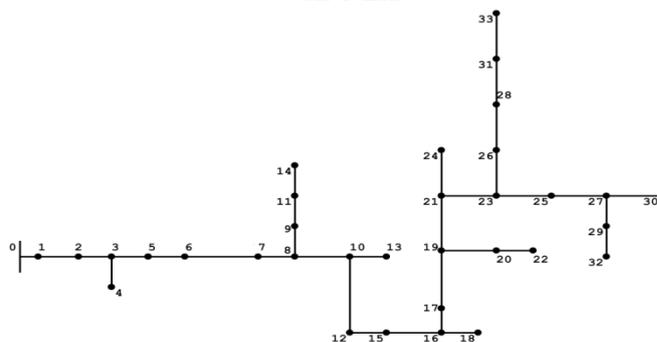


Fig. 12. Power network for the cases tested.