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$\mathcal{H}_2$ Control through Wireless Sensor Network: A trade-off analysis between energy consumption and control performance based on distinct data rates ⋆

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Abstract: This paper investigates the problem of control through a Wireless Sensor Network (WSN). Such communication channel is digital (the system to be controlled must be sampled and its mathematical model is discretized) and is also semi-reliable, since it admits the possibility of packet loss (that can be modeled by means of a Markov chain). The aim is to assure the stability of the closed-loop system and a good controller performance (measured in terms of the $\mathcal{H}_2$ norm) while trying to reduce the energy consumption of the network. For this purpose, an index that relates those conflicting criteria ($\mathcal{H}_2$ norm and energy consumption) is proposed, allowing the designer to choose the best WSN setup. The strategy adopted to develop the trade-off index consists in evaluating different network data rate settings, which implies: distinct sampling rates (better or worse digital representations of the continuous-time plant); distinct energy saving levels (due to higher or lower number of transmissions/receptions per seconds); and distinct probabilities of packet loss (due to, among others, increased network congestion). A numerical example based on the $\mathcal{H}_2$ state-feedback networked control of a Furuta's inverted pendulum is presented to illustrate the employment of the proposed trade-off method. In this example, the statistical data used to model the communication channel are obtained from a WSN workbench.

Keywords: Networked control systems, Semi-reliable communication network, Energy efficiency, Markov jump linear systems, Wireless Network Emulation Workbench.

1. INTRODUCTION

One of the pillars of Industry 4.0 is the unified use of industrial automation and information technology in order to improve efficiency, quality and productivity in manufacturing applications. In this scenario, where wireless sensor networks (WSN) allow the exchange of information among the devices and the use of digital controllers is highly widespread, the implementation of distributed or centralized control systems is enabled by an architecture called networked control system (NCS). Among advantages that support the propagation of the NCS concept in the last decades, one can cite Braga et al. (2014); Wang and Liu (2008): the reduction in system wiring, the use of plug-and-play devices, the increase of system agility, the ease of maintenance and diagnosis, the reduction of the complexity of systems at the price of a nominal economical investment, the lower cost of installation and maintenance than its wired counterparts, the flexible architecture that allows the employment of a multipurpose shared network to connect spatially distributed elements.

Despite the long list of benefits, some disadvantages restrict the employment of the NCS architecture, such as Braga et al. (2014); Wang and Liu (2008); Borges et al. (2010); Lian et al. (2001); Zhang et al. (2001): packet dropouts, multiple-packet transmission, network-induced delays, and band-limited channels. Furthermore, the replacement of a wired communication channel by wireless network faces several challenges, such as choosing the best routing and power constraints. The most challenging issue in WSN is the limited and, many times, unrechargeable energy supply required for data transmission, signal processing, and hardware operation. This concern encourage many research efforts to be directed to the development of energy efficiency strategies.

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Additionally, another obstacle that must be overcome in the design of a NCS is the difficulty to obtain data that properly represent the behavior of the network, since it requires to carry out physical tests in various regions and different situations in order to cover the widest range of obstacles that the network may face. To facilitate the performance of these tests, several researchers (Imran et al., 2010; Fahmy, 2016a,b; Egea-Lopez et al., 2005; Musznicki and Zwierzykowski, 2012; Horneber and Hergenröder, 2014) have proposed analyzing tools, such as simulators, emulators, and testbeds for evaluating WSNs. Simulators are software-based solutions that uses mathematical models to recreate the behavior of wireless communication networks, while emulators run the same code on real platforms. On the other hand, testbeds or workbenches are physical structures that emulate, on smaller scale, the natural phenomena features associated with wireless communication such as, for example, the use of digital attenuators to replicate the transmission signal attenuation resulting from the distance between nodes and distinct obstacles in the environment where the signal could be transmitted (see Delforno (2020) for more details).

After knowing the probabilistic behavior of the network, in order to assure better quality of service while implementing a NCS, the designer must consider to allow a certain performance degradation in the control. For instance, by using lower sampling frequencies in the sensors, implying less faithful representations of the analog systems, however allowing reduce the number of packet transmissions per second (data rate), one can provide a better use of the network: reducing congestion, better management of computing resources and an increase the useful life of the network, thus also impacting the energy costs of transmissions and receptions.

This paper aims to propose an index to evaluate the relationship between the degradation of control quality and the energy savings provided by the reduction of the transmission rate in the wireless network. In this sense, after carrying out tests on an emulation workbench, the probabilistic information associated with packet loss for different network data rate setups is employed to obtain a Markovian model for the wireless communication channel. Such a model is based on two modes of operation: one representing failure and the other one representing successful transmissions. The jumps between the Markov chain states are governed by a transition probability matrix with a Bernoulli distribution. Knowing the size of packets transmitted by the tested WSN, it is possible to determine the maximum sampling period for the different transmission rate settings (in kilobits per second), in order to obtain a discrete approximation for the continuous model that represents a real world plant. Based on the theory of Markov jump linear systems (MJLS), which can properly represent the abrupt transition between the failure and success modes of transmission in NCS, a stochastic stabilizing controller for the discrete-time Markov model of the plant is designed, while minimizing a performance criterion commonly used in control: the $\mathcal{H}_2$ norm. Then, although the remote access to the workbench does not allow the measurement of the energy consumption of the network, a rough estimate is obtained from an energy computation model presented in the literature. Finally, as main contribution of this paper, a trade-off index, that allows the calculation of the cost-benefit ratio between controller performance (measured in terms of the $\mathcal{H}_2$ norm) and the energy consumption of the network nodes, is proposed. Such index helps choosing the most suitable network setup for the design purposes. An illustrative example of obtaining the index is presented for the networked control of Furuta pendulum.

Notation: Capital and lowercase letters respectively denote matrices and vectors (or scalars). The set of real numbers is denoted by $\mathbb{R}$. For real matrices and vectors the transposition is indicated by the $(')$, while, in square matrices, $(\star)$ is used for blocks induced by symmetry and $\mathrm{Tr}(X)$ represents the trace function of Matrix X. $\mathbb{K} = \{1, \ldots, \sigma\}$ is the finite set with σ entries. The fundamental probability space is represented by $(\Omega, \mathcal{F}, \{\mathcal{F}_k\}, \Gamma)$, while $\{\theta_k; k \geq 0\}$ represents the discrete-time homogeneous (time-invariant) Markov chain, having $\Gamma = [p_{ij}], \forall i, j \in \mathbb{K}$, as the transition probability matrix, where

$$p_{ij} = \Pr(\theta_{k+1} = j | \theta_k = i), \qquad k \geq 0. \tag{1}$$

The initial probability distribution is given by $\mu = [\mu_1, \ldots, \mu_\sigma]$, where $\mu_i = \Pr(\theta_0 = i)$. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation. The Hilbert space formed by the stochastic process $z = z(k); k \geq 0 \in \mathbb{R}^m$, such that $\|z\|_2^2 = \sum_{k=0}^{\infty} = \mathbb{E}\{z(k)'z(k)\}$ is denoted by ℓ_2^m.

2. FUNDAMENTALS ON $\mathcal{H}_2$ STATE-FEEDBACK CONTROL OF MARKOV JUMP LINEAR SYSTEMS

Knowing that Markov jump linear system (MJLS) is a class of stochastic systems with linear operation modes that can model processes that suffer abrupt changes in their operation point, such as, packet loss in NCS, consider a discrete-time MJLS represented by

$$\mathcal{G} = \begin{cases} x(k+1) = A_{\theta_k} x(k) + B_{\theta_k} u(k) + J_{\theta_k} w(k), \\ \quad\;\; z(k) = C_{\theta_k} x(k) + D_{\theta_k} u(k) + E_{\theta_k} w(k) \end{cases} \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$, $z(k) \in \mathbb{R}^{n_z}$ and $w(k) \in \mathbb{R}^{n_w}$ respectively represent the state, control input, controlled output and external perturbation vectors. Also consider that the sub-index [1] θ_k of matrices A, B, J, C, D, E is a random variable that assumes values on the finite set $\mathbb{K} = \{1, 2 \ldots, \sigma\}$ composed by the σ linear operation modes of the MJLS whose transition is governed by a transition probability matrix $\Gamma = [p_{ij}]$, with p_{ij} given by (1). This paper consider a generalized Bernoulli probability distribution, that is, $p_{ij} = p_j, \forall i \in \mathbb{K}$. Regarding the control input, a mode-independent state-feedback control law is considered

$$u(k) = Kx(k) \tag{3}$$

such that the closed-loop MJLS is given by

$$\mathcal{G}_{cl} = \begin{cases} x(k+1) = (A_{\theta_k} + B_{\theta_k} K)\, x(k) + J_{\theta_k} w(k), \\ \quad\;\; z(k) = (C_{\theta_k} + D_{\theta_k} K)\, x(k) + E_{\theta_k} w(k). \end{cases} \tag{4}$$

The control design must assure the mean square stability (MSS, that is, $\mathbb{E}[\|x(k)\|] \to 0$ when $k \to \infty$ for any initial condition $x(0) \in \mathbb{R}^{n_x}$ and $\theta_0 \in \mathbb{K}$ (Costa et al., 2005)) at the same time that minimizes the $\mathcal{H}_2$ norm of the closed-loop MJLS. The $\mathcal{H}_2$ norm is a performance index that indicates how much control effort is required to bring the

[1] To ease the notation, whenever $\theta_k = i, \forall i \in \mathbb{K}$ in $A_{\theta_k}, B_{\theta_k}$ and so forth.

system from an initial condition to stability, in this sense, by minimizing the $\mathcal{H}_2$ norm it is possible to provide a control system that consumes less energy in the actuator. The formal definition for the $\mathcal{H}_2$ norm of an MJLS is presented next (do Val et al., 2002).

Definition 1. Suppose that (4) is MSS with null initial condition ($x(0) = 0$), initial probability distribution given by μ, and $w(k)$ equal to a discrete-time impulse, then the $\mathcal{H}_2$ norm of (4) is defined as

$$\|\mathcal{G}_{cl}\|_2^2 = \sum_{s=1}^{n_w} \sum_{i=1}^{\sigma} \mu_i \|z(k)^{s,i}\|_2^2. \tag{5}$$

Finally, an adaptation for the mode-independent case of the necessary and sufficient LMI conditions for the optimal $\mathcal{H}_2$ state-feedback control of MJLS (2) with generalized Bernoulli distribution proposed in Fioravanti et al. (2014), is presented next.

Lemma 1. (Adapted from Fioravanti et al. (2014)). There exists a controller of the form (3) such that $\|\mathcal{G}_{cl}\|_2^2 < \rho$, under assumption $p_{ij} = p_j, \forall i, j \in \mathbb{K}$ for the transition probabilities, if and only if there exist symmetric matrices Z_i, X, and Y of compatible dimensions satisfying $\sum_{i\in\mathbb{K}} \mu_i \mathrm{Tr}(W_i) < \rho$ and the LMIs

$$\begin{bmatrix} W_i & \star & \star \\ J_i & X & \star \\ E_i & 0 & I \end{bmatrix} > 0, \tag{6}$$

$$\begin{bmatrix} Z_i & \star & \star \\ A_iX + B_iY & X & \star \\ C_iX + D_iY & 0 & I \end{bmatrix} > 0, \tag{7}$$

$$\sum_{i\in\mathbb{K}} (p_i Z_i) - X < 0 \tag{8}$$

for all $i \in \mathbb{K}$. In the affirmative case, the mode-independent state feedback gain is given by $K = YX^{-1}$.

3. NETWORK WORKBENCH

In order to obtain network parameters for modeling, stability analysis and control in dynamical closed-loop systems where the elements (controllers, actuators, sensors) communicate by a point-point link, a user-friendly platform of wireless network simulation (Egea-Lopez et al., 2005; Musznicki and Zwierzykowski, 2012; Imran et al., 2010; Fahmy, 2016a), emulation or testbed (Imran et al., 2010; Fahmy, 2016a,b; Horneber and Hergenröder, 2014) would be helpful. The wireless network workbench used in this paper was proposed in[2] Delforno (2020). It allows to emulate the behavior of a two-node low-power network under different operating conditions. Besides, it is useful to evaluate, among others, system losses, transmission power setups, modulations, carrier frequency, channel spacing, attenuation behavior (as investigated in Serafini et al. (2021)), or data rate setups, providing some statistical data that can be employed in NCS design.

The open-source nature of this workbench ensures full control over the firmware, software, and hardware, that is, any radio communication solution can be employed. However, the current setup is constituted by two radio communication modules BE900 (RADIOIT ELETRONICA LTDA, 2012) operating at the Industrial, Scientific, and Medical (ISM) frequency of 915 MHz and uses the Radiuino Platform (Branquinho et al., 2010). In summary, the network workbench shown in Figure 1 was assembled as follows: the sensor nodes are connected by coaxial cables in order to confine the radio frequency signal, ensuring an ideal propagation without external interference. All attenuations that could result from an actual wireless propagation were represented by mathematical models that were validated after a series of field tests. The resulting attenuation levels are emulated by a microcontroller module that controls a variable attenuator placed between the sink and source nodes.

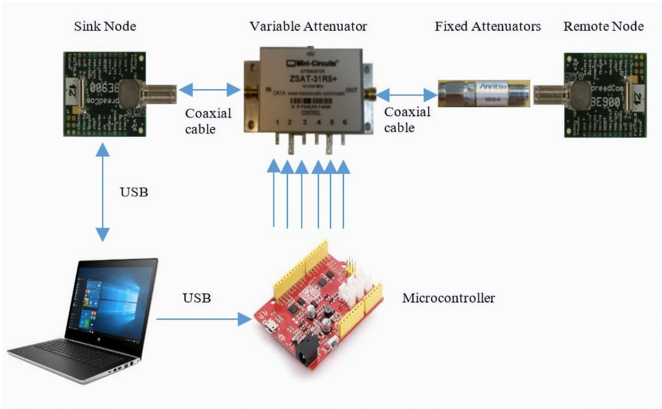

Figure 1. The workbench components and connection setup.

In the present paper, the workbench was used to collect packet loss statistical information for different data rate settings. Such data is used in the Markov chain model that represents the communication channel between controller and plant. Assuming a Bernoulli distribution, and an MJLS with two operation modes: 1 - representing a successful transmission and 2 - failure, the transition probability matrix is given by

$$\Gamma = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} (1-PER) & PER \\ (1-PER) & PER \end{bmatrix} \tag{9}$$

where PER is the packet error rate provided by the Human-Machine-Interface (HMI) that controls the workbench operation, as shown in Figure 2 for a data rate of 250 kbps.

Additionally, as the communication channel is digital, even if the controlled plant is analog, the discrete-time equivalent of the system must be computed so that the controller can be implemented digitally. In this sense, different data rate settings would allow different sampling rates. The network workbench is configured to operate with the following data rates: $\{4.8, 10, 38.4, 250\}kbps$ which were chosen from a list of preferred settings for a wide range of data rates provided in Lunder (2007). Knowing that the size of each packet sent over the emulated network is 52 bytes (or 416 bits) using the Radiuino Platform, for a particular data rate it is possible to determine the maximum number of packets that can be transmitted per second (N_{max}) by calculating the ratio between data rate and packet size (416 bits). For example, for 4.8 kbps, $N_{max} = 4800/416 = 11.5385$ packets per second. Thus,

[2] See Serafini et al. (2021) for a more detailed description about the network workbench employed in this paper

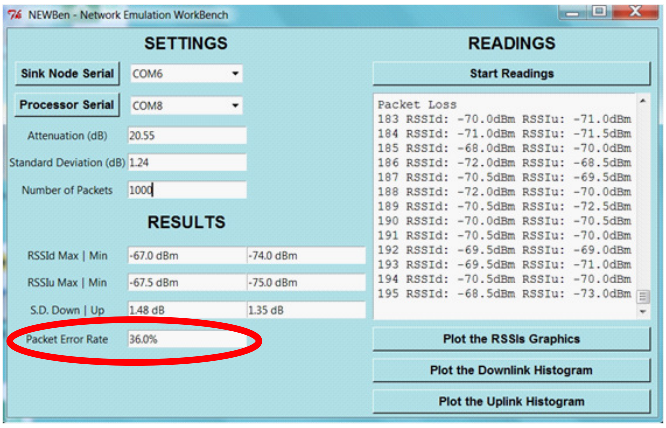

Figure 2. The HMI showing the settings and results menus. The obtained PER for 250 Kbps is circled in red.

in the controller design stage, this value would serve as an upper bound for the sampling frequency ($f \leq N_{max}$) and, consequently, it would be possible to determine the minimum sampling period ($T = 1/f$) used to discretize the dynamic system associated with each data rate.

4. ENERGY CONSUMPTION FOR HOP-BY-HOP NETWORKS

As the tests on the emulation workbench were performed remotely, it was not possible to measure the energy consumption in practice. However, there are models in the literature for calculating energy consumption in more complex networks (such as multi-hop networks with hop-by-hop transport schemes), which include the two-nodes emulated network as a particular case. In this article, it was adopted the formula provided in (Palma et al., 2018, Eq.(17)):

$$\mathcal{E}(E_T(p, L, N)) = \Xi \times \frac{(E_{TX} + 2E_{SW} + pE_{RX})\,\mathbb{E}(M)}{T_s} \tag{10}$$

where[3] E_{TX}, E_{RX} and E_{SW} are, respectively, the energy required to transmit a packet, receive a packet, and switch the network from transmission mode to receive mode or vice versa, T_s is the sampling time in seconds (associated with the data rate, as discussed in the previous section), Ξ is a correction factor to convert the time units of the energy consumption, in this case to convert from seconds to hours $\Xi = 3600$ is adopted. Additionally, $\mathbb{E}(M)$ is the mathematical expectation of the global number of transmissions calculated as below

$$\mathbb{E}(M) = \frac{\left[1-(1-p^2)^L\right]}{p^2(1+p)^{-1}} \cdot \frac{1-\left[1-(1-p)^L\right]^N}{(1+p)^L} \tag{11}$$

where p represents the probability of a successful transmission ($p = 1 - PER$, and PER is the packet error rate provided by the workbench), L is the maximum number of retransmissions (in this paper it is considered that each packet is sent a single time, that is, $L = 1$), N is the number of hops between the source and sink (in the case of a two-nodes network, a single hop is required, that is, $N = 1$).

[3] The values E_{TX}, E_{SW} and E_{RX} used in the example were taken from [12].

5. TRADE-OFF INDEX

In order to assist in determine the most suitable network setup for the control project, this section proposes a trade-off index $\mathcal{J}$ that relates the percentage of energy saved and the percentage of degradation in the control strategy, measured in terms of the $\mathcal{H}_2$ norm. For this purpose, it is first required the computation of index $\mathfrak{H}_i$ that corresponds to the percentage improvement of the $\mathcal{H}_2$ norm, given by:

$$\mathfrak{H}_i = 100 \times \frac{\rho_{max} - \rho_i}{\rho_{max}}\% \tag{12}$$

where[4] ρ_i is the $\mathcal{H}_2$ guaranteed cost obtained from Lemma 1 and the sub-index $i = 1, \ldots, n$ represents the n distinct network setups (in this case, each network setup is associated with a different data rate). Then, it is necessary to compute index $\mathfrak{E}_i$ representing the percentage improvement in energy savings, given by:

$$\mathfrak{E}_i = 100 \times \frac{\mathcal{E}_{max} - \mathcal{E}_i}{\mathcal{E}_{max}}\% \tag{13}$$

where[5] $\mathcal{E}_i$ represents the network energy consumption for each network setup, measured in Joules/hour computed in (10). And finally, it is proposed the index $\mathcal{J}_i$, which represents how much the two previously defined indexes simultaneously contribute to the choice of the network setup. The trade-off index $\mathcal{J}_i$ is provided by the following equation

$$\mathcal{J}_i = a_1 \cdot \mathfrak{H}_i + a_2 \cdot \mathfrak{E}_i, \qquad a_1 + a_2 = 1, \qquad a_1, a_2 \geq 0 \tag{14}$$

where a_1 and a_2 represent the weight or importance of each one of the indexes ($\mathfrak{H}_i$ and $\mathfrak{E}_i$) for the final computation. Therefore, if the designer is interested in greater energy savings, he should assign greater importance to index $\mathfrak{E}_i$, that is, to impose $a_2 \geq 50\%$. On the other hand, if control performance is the designer priority, the factor a_1 must be incremented, increasing the importance of the portion $\mathfrak{H}_i$. In this work, aiming that both terms have the same relevance in the final trade-off index, $a_1 = a_2 = 0.5$ are adopted. Note that the choice of network setup that ensures the most advantageous control design, is given by the sub-index i associated with the highest $\mathcal{J}$ value, because the higher the value of $\mathcal{J}$, the greater the cost-benefit, that is, the network setup i simultaneously produces a reasonable reduction in energy consumption without penalizing the control effort.

6. CASE STUDY: FURUTA PENDULUM

This section investigates the relationship between the energy consumption of a WSN configured with different data rates and the control performance measured in terms of the $\mathcal{H}_2$ norm for a state-feedback control design through semi-reliable network. For this purpose, the mathematical model of a rotational inverted pendulum (Cazzolato and Prime, 2011; Qua, 2017) linearized around its unstable equilibrium point is considered. The state space realization for the continuous-time plant obtained by Oliveira (2015) is given below

$$\mathcal{G}_c = \begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) + J_c w(t), \\ z(t) = C_c x(t) + D_c u(t) + E_c w(t), \quad t \in \mathbb{R} \end{cases} \tag{15}$$

[4] $\rho_{max} = \max_{i=1,\ldots,n} \rho_i$
[5] $\mathcal{E}_{max} = \max_{i=1,\ldots,n} \mathcal{E}_i$

Table 1. Relation between different WSN setups associated with distinct data rates used in Radiunio transmission; maximum number of packets transmitted by second (N_{max}); chosen sampling frequency ($f < N_{max}$); adopted sampling interval (T); packet error rate ($PER(\%)$); $\mathcal{H}_2$ performance index (ρ); and energy consumption ($\mathcal{E}$).

Setup	Data rate (kbps)	$N_{max}(Hz)$	$f(Hz)$	$T(ms)$	$PER(\%)$	ρ	$\mathcal{E}(J/h)$
1	4.8	11.5	10	100	1	1.11	32.97
2	10	24	20	50	4	0.58	64.23
3	38.4	92.3	80	12.5	17	0.17	228.08
4	250	600.9	500	2	36	0.03	1181

where

$$A_c = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 34.16 & -18.62 & -0.035 \\ 0 & 76.74 & -17.96 & -0.079 \end{bmatrix}, \; B_c = \begin{bmatrix} 0 \\ 0 \\ 18.31 \\ 17.65 \end{bmatrix} \tag{16}$$

and supposing that the noise can achieve 50% of the control signal level, one has $J_c = 0.5B_c$. Furthermore, assuming that the controlled output is equal the state vector ($z(t) = x(t)$), the output matrices are $C_c = I$, $D_c = 0$ and $E_c = 0$. Since the goal is to control the plant using a digital communication network, the first step is to obtain a discrete-time representation of (15) by employing zero order hold technique with sampling time given by the computations presented in the last paragraph of Section 3 to obtain the discrete-time state space matrices A_d, B_d, J_d, C_d, D_d, E_d from A_c, B_c, J_c, C_c, D_c, E_c. The second step is to define a communication model that considers the failures, in this case the Zero-input approach (Schenato, 2009) is adopted. In summary, supposing a failure in the communication link between plant and controller, the controlled system is represented by the MJLS (2) with two operation modes: $\theta_k = 1$ representing successful transmission where the state space matrices have nominal value ($A_1 = A_d$, $B_1 = B_d$, $J_1 = J_d$, $C_1 = C_d$, $D_1 = D_d$, $E_1 = E_d$); and $\theta_k = 2$ representing failure meaning that the control signal is lost, that is, the control matrices are null ($A_2 = A_d$, $B_2 = 0$, $J_2 = J_d$, $C_2 = C_d$, $D_2 = 0$, $E_2 = E_d$). As previously mentioned, the transition between the modes obeys a generalized Bernoulli distribution, such that Γ is provided by (9). Also, assuming that the communication starts from a successful transmission, the initial probability distribution is given by $\mu = [1\ 0]$.

Finally, following the instructions presented in Section 3, one can get: maximum number of packets that can be transmitted by second (N_{max}); the sampling frequency ($f < N_{max}$); the minimum sampling interval ($T = 1/f$) and the packet error rate ($PER(\%)$) for each value of data rate available for tests in the emulation workbench (4.8, 10, 38.4, 205 kbps). Those parameters and the results regarding control performance ($\mathcal{H}_2$ norm provided by Lemma 1) and energy consumption ($\mathcal{E}$ computed in (10)) obtained for each data rate are presented in Table 1 and graphically displayed in Figure 3.

To conclude about the best network configuration, in which energy savings compensate the performance degradation in the control or vice versa, the results from Table 1 are used to calculate the indices that indicate the percentage improvement in the $\mathcal{H}_2$ norm ($\mathfrak{H}$), the percentage improvement in energy savings ($\mathfrak{E}$) and the best trade-off ($\mathcal{J}$) by employing equations (12), (13) and (14), respectively. The values of the indices are graphically illustrated in Figure 4. Note that, although the behavior of indices $\mathfrak{H}$ and $\mathfrak{E}$ is monotonic with respect to data rate transmission [6], the relationship between them produced by $\mathcal{J}$ demonstrates that an intermediate transmission rate (38.4kbps) can be more advantageous.

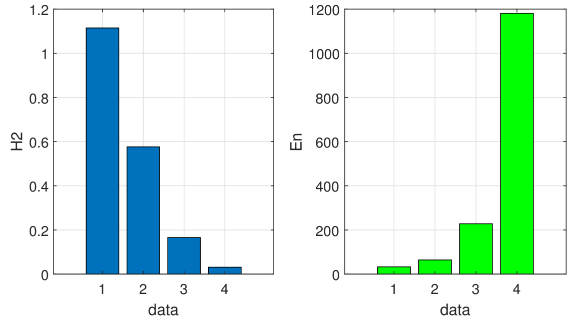

Figure 3. $\mathcal{H}_2$ performance index (ρ) obtained from Lemma 1 in blue, Energy consumption ($\mathcal{E}$) provided by Eq. (10) in green for each WSN setup described in Table 1.

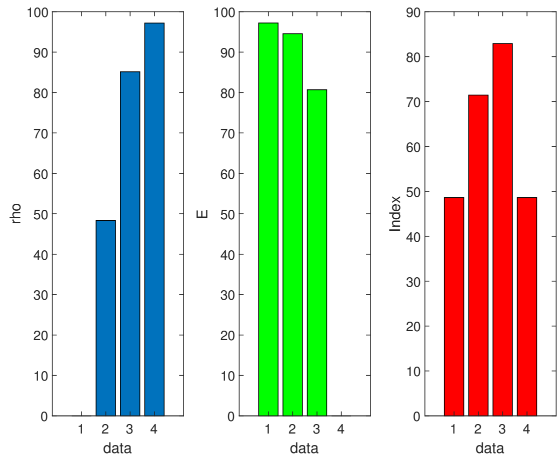

Figure 4. Percentage improvement in the $\mathcal{H}_2$ norm ($\mathfrak{H}$ from Eq. (12) in blue), percentage improvement in energy savings ($\mathfrak{E}$ from Eq. (13) in green) and the best trade-off ($\mathcal{J}$ from Eq. (14) with $a_1 = a_2 = 0.5$ in red) for each WSN setup described in Table 1.

[6] Figure 4 also shows that the percentage improvement of ρ and $\mathcal{E}$ (computed respectively by (12) and (13)) is null in the worst case network setup, since Setup=1 (4.8kbps) provides $\rho_i = \rho_{max}$ and Setup=4 (250kbps) provides $\mathcal{E}_i = \mathcal{E}_{max}$.

7. CONCLUSION

The obtained results have shown that by augmenting the data rate from 4.8 Kbps to 250 Kbps, there is a significant increase in the packet dropout (indicated by PER), implying that as faster the data transmission, less reliable it is. Nevertheless, communication channel settled with higher data rates allow lower sampling intervals, which produce a more accurate digital representation of the continuous-time behavior of the dynamic system, allowing to design an optimized controller (associated with a better $\mathcal{H}_2$ performance). At the same time, higher data rates are associated with an augmentation of the energy consumption. Therefore, in extreme cases, or a system that saves energy but has poor control performance (economy mode) will be implemented, or a system that uses a lot of energy and has good control performance (optimized control mode) will be implemented. For this reason, it is important to develop a method that helps the designers to choose the network setup best suited to their purposes, taking into account the trade-off between control quality and energy efficiency. To obtain a good cost-benefit ratio, the solution proposed in this paper is the computation of an index $\mathcal{J}$ that identifies the best network setup relating the percentage of $\mathcal{H}_2$ norm degradation and the percentage savings in energy consumption. After creating this index, the results indicated that the best network setup for a particular example was obtained for an intermediate data rate, that guarantees control with a reasonable good performance and simultaneously allows some economy in the network energy consumption. In future works, the authors intended to carry out experimental measurements of energy consumption on the workbench, in addition to developing a new set of firmware that allow performing communication tests for different data rate configurations.

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