Robust PID Controller for Second-Order Systems plus Time Delay

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Abstract: This paper presents a design methodology for obtaining a robust proportional-integral-derivative (PID) multivariable controller for second-order linear systems with time-varying delay, guaranteeing a pre-established exponential decay rate. Relevant control challenges such as stabilization, modeling error and constant reference tracking are addressed within the proposed Linear Matrix Inequality (LMI) design approach. The design strategy is derived from a transformation that can be applied to obtain constant reference tracking for an actuated subspace of underacted systems. Furthermore, the integral action has an additional objective which is to increase the degree of design. Simulation case studies are used to highlight the benefits of the proposed results.

Resumo: Este artigo apresenta uma metodologia de projeto para a obtenção de um controlador multivariável proporcional-integral-derivativo (PID) robusto para sistemas lineares de segunda-ordem com atraso variante no tempo, garantindo uma taxa de decaimento exponencial pré-estabelecida. Desafios de controle relevantes, como estabilização, erro de modelagem e rastreamento de referência constante são tratados dentro da proposta de abordagem de projeto de Desigualdade de Matriz Linear (LMI). A estratégia de projeto é obtida a partir de uma transformação que pode ser aplicada para obter rastreamento de referência constante para um subespaço atuado de sistemas subatuados. Além disso, a ação integral tem um objetivo adicional que é aumentar o grau de liberdade de projeto. Estudos de caso de simulação são usados para destacar os benefícios dos resultados propostos.

Keywords: Robust Control, Time Delay, Linear Matrix Inequalities, Second-Order Systems, Proportional-Integral-Derivative controller (PID)

Palavras-chaves: Controle Robusto, Atraso no Tempo, Desigualdades Matriciais Lineares, Sistemas de Segunda-Ordem, Controlador Proporcional-Integral-Derivativo (PID)

1. INTRODUCTION

The field of vibration mechanics is of paramount importance and has originated several contributions from researchers in recent years, since in some processes such vibrations can be unwanted, compromising the performance of machines and structures (Hiramoto and Grigoriadis, 2016; Zhang et al., 2021). Vibratory systems may be subject to time delay, due to many reasons, such as detection and actuation in the feedback of states, physical separation between sensors and measurement points, delay in communication channels, online data acquisition, filtering, signal transmission from a computer to the actuator, which can degrade control performance and destabilize a system (Araújo and Santos, 2018; Zhang et al., 2020). In general, time-delayed second-order systems can be stable in open-loop and become unstable in closed-loop if delays are not properly considered at some stage of the control design. In this context, delays can be constant or time-varying. Because the notion of poles or eigenvalues cannot be applied to systems subject to time-varying delays (Santos et al., 2018) to design controllers for this class of system is more complex. However, because time-varying delay are found in a large class of systems (Yu et al., 2015; Seguy et al., 2010), it is important to develop control design methodologies for systems subject to time-varying delays.

The literature discusses the constant and time-varying delay in second-order systems (Araújo and Santos, 2018; Santos et al., 2018). In Araújo and Santos (2018) a method based on reception and smith predictor is proposed to handle the constant delay, however, the proposal deals only with stable and marginally stable systems, not guaranteeing the internal stability of unstable open-loop systems. Araujo and Santos (2020) proposes a sample data strategy

⋆ This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES), Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brasil (CNPq), and Fundação de Amparo à Pesquisa do Estado de Minas Gerais - Brasil (FAPEMIG).

to apply a smith predictor based approach to solve these unstable problems.

Regarding constant delays, Natori et al. (2008) investigates a dedicated control scheme for delay compensation based on the concept of network disturbance and communication disturbance observer. Belotti and Richiedei (2020) proposes a numerical method for partial poles placement in second-order systems, which allows a priori verification of the stability of primary and secondary roots through the reception method, which combined with an LMI condition, guarantees asymptotic stability for a given delay upper bound.

More recent studies have focused on the treatment of systems subject to time-varying delays, using robust control techniques. The use of LMIs and the Lyapunov-Krasovskii theory is an effective methodology to investigate the stability of time-varying delay systems, this methodology is addressed in this paper.

Motivated by the problem of controlling time-varying delay systems, this paper presents a robust framework for designing multivariable Proportional-Integral-Derivative (PID) controllers for second-order systems via LMI formulations, guaranteeing a rate pre-established convergence, thus improving transient performance. In contrast to related work based only on proportional and derivative feedback, integral action is used to achieve null tracking error for constant set-points concerning the actuated states and to increase design flexibility.

2. PROBLEM FORMULATION

Consider the class of systems described by the second-order linear model:

$$M\ddot{z}(t) + D\dot{z}(t) + Sz(t) = Bu(t - d(t)) \tag{1}$$

where $z(t) \in \mathbb{R}^n$ is the state vector and $w(t) \in \mathbb{R}^p$ is the exogenous disturbance vector, the delay $d(t)$ affects the control signal $u(t - d(t))$. The delay is modeled as $d(t) = \tau + \mu(t)$, representing a time-varying delay, with τ being the nominal delay value and $\mu(t)$ a time-varying scalar function, which satisfies $\mu(t) \leq |\mu(t)| \leq \tau$. M, D, $S \in \mathbb{R}^{n\times n}$ are, respectively, the mass, damping and stiffness matrices, and $B \in \mathbb{R}^{n\times m}$ is the control matrix. In this paper we assume that the mass matrix M is nonsingular.

The control signal $u(t)$ is an extension of the single-variable PID controller to the multivariable case, as so the proportional control action (P) is proportional to the current error. The integral action (I) can eliminate the steady-state offset and the derivative action (D) is specially related with shaping the damping behavior of the closed-loop system. Thus the multivariable PID controller is defined by

$$u(t) = K_P e(t) + K_I \int_0^t e_a(\tau)d\tau + K_D\dot{e}(t), \tag{2}$$

with $e(t) = z(t) - r(t)$ where $r(t) \in \mathbb{R}^n$ is the desired set-point vector, and $e_a(t)$ denotes the error signal on the actuated states; in the case of full actuated systems $e_a(t) = e(t)$, otherwise, $e_a(t) = Ue(t) \in \mathbb{R}^a$, for an appropriated binary matrix $U \in \mathbb{R}^{a\times n}$, such that $e_a(t)$ is composed only by the entries of $e(t)$ resulting from the actuated states. In addition $K_P, K_D \in \mathbb{R}^{m\times n}$ and $K_I \in \mathbb{R}^{m\times a}$. Notice that we cannot guarantee perfect tracking of arbitrary constant reference in the underactuated case, due to the control action limitation. Thus the actuated error, namely $e_a(t)$, is defined in order to ensure trajectory tracking on the actuated degrees-of-freedom.

In order to enjoy from the LMI framework to design the robust controller we rewrite the system (1) such that the PID part of the controller (2) becomes a static state feedback controller. In view of that we define the following state variables:

$$x_1(t) = z(t), \quad x_2(t) = \dot{z}(t), \quad x_3(t) = -\int_0^t e_a(\tau)d\tau \tag{3}$$

thus the closed-loop descriptor system is given as

$$\begin{cases} E\dot{x}(t) = Ax(t) + B_u u(t - d(t)) & (4.1) \\ \qquad + ([0 \quad 0 \quad -U]^T)r(t) & \\ u(t - d(t)) = Kx(t - d(t)) & \\ \qquad - K[r^T(t) \ \ \dot{r}^T(t) \ \ 0_{1\times a}]^T & (4.2) \\ y(t) = Cx(t) & (4.3) \end{cases}$$

where

$$x^T(t) = [x_1^T(t) \ \ x_2^T(t) \ \ x_3^T(t)],$$

$$A = \begin{bmatrix} 0_n & I_n & 0_{n\times a} \\ -S & -D & 0_{n\times a} \\ U & 0_{a\times n} & 0_a \end{bmatrix}, \quad B_u = \begin{bmatrix} 0_{n\times m} \\ B \\ 0_{a\times m} \end{bmatrix},$$

$$E = \mathrm{diag}\{I_n, M, \ I_a\},$$

$$K = [K_{P m\times n} \ K_{D m\times n} \ K_{I m\times a}], \quad C = [I_n \ 0_n \ 0_{n\times a}].$$

Then replacing (4.2) in (4.1), we get:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t - d(t)) & (5.1) \\ \qquad + B_u K([r^T(t) \quad \dot{r}^T(t) \ 0]^T) & \\ \qquad + ([0 \quad 0 \quad -U]^T)r(t) & \\ y(t) = Cx(t) & (5.2) \end{cases}$$

where $A_d = B_u K$, doing this transformation we get a state feedback project, being easily solved via LMIs.

3. MAIN RESULTS

In this section, appropriate conditions are proposed for the state feedback design of the system (5) subject to time-varying delay. The control problem can be stated as follows.

Problem 1. Design a PID controller for the time-delayed second-order linear system in (1) ensuring stability and a given exponential decay rate for the closed-loop system.

3.1 Design conditions

Next, we present the LMI conditions for solving Problem 1 by guaranteeing a pre-specified exponential convergence rate.

Theorem 1. Let $\tau > 0, \tau \geq \mu \geq 0$, such that $d(t) \in [\tau - \mu, \tau + \mu]$, $\delta > 0$ and $\alpha \neq 0$ a scalar fit parameter. Then, the system (5) is exponentially stabilized with exponential convergence rate δ by the PID controller with gains given in the matrix $K = X\overline{F}^{-T}$, if there exist matrices of appropriate dimensions: $\overline{F}$, $\overline{P} = \overline{P}^T$, $\overline{S} = \overline{S}^T$, $\overline{Q}$, $\overline{R}_1 = \overline{R}_1^T$, $\overline{R}_2$, $\overline{R}_3 = \overline{R}_3^T$, $\overline{Z} = \overline{Z}^T$ and X such that the following LMIs are satisfied

$$\begin{bmatrix} \overline{P} & \star \\ \overline{Q}^T & \varepsilon_1 \overline{S} \end{bmatrix} > 0, \tag{6}$$

where $\varepsilon_1 = e^{-2\delta\tau}/\tau$,

$$\overline{R} = \begin{bmatrix} \overline{R}_1 & \star \\ \overline{R}_2 & \overline{R}_3 \end{bmatrix} > 0, \tag{7}$$

and

$$\left[\begin{array}{c|c} \overline{\Xi} & \star \\ \hline \overline{\Gamma}^T & \varepsilon_2^{-1}\mu\overline{Z} \end{array}\right] < 0, \tag{8}$$

where $\varepsilon_2 = e^{-2\delta(\tau+\mu)}$, $\overline{\Gamma}^T = \mu[X^T B^T \;\; \alpha X^T B^T \;\; 0 \;\; 0]$ and $\overline{\Xi}$ give in (A.4).

$$\overline{\Xi} = \begin{bmatrix} \overline{\mathcal{F}} & \star & \star & \star \\ E(\overline{P} + \tau\overline{R}_2)E^T - \varepsilon_2(E\overline{F}E^T - \alpha A\overline{F}^T E^T) & \star & \star & E\mathcal{G}E^T \\ \varepsilon_1 \overline{R}_3^T E^T - \overline{Q}^T E^T + \varepsilon_2 X^T B^T & \varepsilon_2 \alpha X^T B^T & -\varepsilon_1(\overline{R}_3 + \tau\overline{S}) & \star \\ 2\delta\overline{Q}^T E^T - \varepsilon_1 \overline{R}_2^T E^T & \overline{Q}^T E^T & \varepsilon_1 \overline{R}_2^T & -\varepsilon_1 \overline{R}_1 \end{bmatrix}. \tag{9}$$

where $\overline{\mathcal{F}} = E(2\delta\overline{P} + \overline{Q} + \overline{Q}^T + \tau\overline{R}_1 - \epsilon_1\overline{R}_3 + \overline{S})E^T + \varepsilon_2(A\overline{F}^T E^T + E\overline{F}A^T)$ and $\mathcal{G} = \tau R_3 + 2\mu\overline{Z} - \epsilon_2(\overline{G} + \overline{G}^T)$.

Proof 1. Since we assumed that the mass matrix M is nonsingular the matrix E in (5.1) is nonsingular as well. Then left-multiplying (5.1) by E^{-1} the descriptor system (5) becomes a standard space-state description. Then we apply Lemma 1, Appendix A, to the resulting system description.

Now defining the variables: $\overline{F} \triangleq F^{-1}$ and

$$[\overline{P} \;\; \overline{Q} \;\; \overline{R}_1 \;\; \overline{R}_2 \;\; \overline{R}_3 \;\; \overline{Z}] \triangleq \overline{F}[P \;\; Q \;\; R_1 \;\; R_2 \;\; R_3 \;\; Z]\overline{F}^T$$

The LMIs in (6) and (7) are obtained pre- and post-multiplying the LMIs (A.1) and (A.2), respectively, by $diag\{\overline{F}, \ldots, \overline{F}\}$ and $diag\{\overline{F}, \ldots, \overline{F}\}^T$.

Furthermore, the LMI in (8) is obtained through the LMI in (A.3) performing the substitutions: $A_d = E^{-1}BK$, $A = E^{-1}A$ and $G = \alpha F$ in (8), and pre- and post- multiplying (8) by $diag\{\overline{F}, \ldots, \overline{F}\}$ and $diag\{\overline{F}, \ldots, \overline{F}\}^T$. Finally, considering the new linearizing variable $X = K\overline{F}^T$ and pre- and post-multiplying the LMI (8) by $diag\ \{E, E, I, I\}$ and its transpose, respectively, the conditions in the Theorem 1 are obtained.

Procedure 1. PID controller design for solving Problem 1:

Step 1: Rewrite the time-delayed second-order linear system (1) as the augmented descriptor model (5.1);

Step 2: Define the decay rate of the system response, $\delta > 0$, and α, a free tuning parameter, defining the variation in the value of the delay, μ.

Step 3: Find the solution $(X, \overline{F})$ that solves the LMI conditions presented in Theorem 1.

Step 4: The PID controller parameters are given in $K = [K_P \;\; K_D \;\; K_I]$, where $K = X\overline{F}^{-T}$.

4. NUMERICAL EXAMPLES

In this section, are presented three numerical examples drawn from the literature to illustrate and validate the proposed robust PID controller design method. In all case studies, a time-varying delay was considered, expressed in (1) by $d(t)$. This term is a time-varying function within the interval $[\tau - \mu, \tau + \mu]$, where τ is the nominal value of the delay and μ the lower and upper bound imposed on the variation of the delay. It was considered $d(t)$ as a random function as illustrated in Fig. 1.

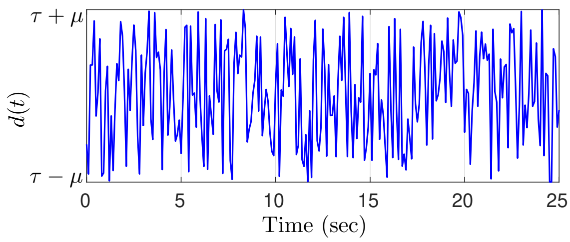

Figura 1. Time-varying delay given by a random function $d(t)$.

4.1 Example 1

In this example a standard benchmark is considered, the 3-DoF model for a wing in an airflow studied in Araujo and Santos (2020), the system matrices are given by:

$$M = \begin{bmatrix} 17.6 & 1.28 & 2.89 \\ 1.28 & 0.824 & 0.413 \\ 2.89 & 0.413 & 0.725 \end{bmatrix}, \quad C = \begin{bmatrix} 7.66 & 2.5 & 2.1 \\ 0.23 & 1.04 & 0.223 \\ 0.6 & 0.756 & 0.658 \end{bmatrix},$$

$$K = \begin{bmatrix} 121 & 18.9 & 15.9 \\ 0 & 27 & 0.145 \\ 11.9 & 3.64 & 15.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad U = B^T. \tag{10}$$

It is important to note that the system is unstable in open loop. The delay value considered is $\tau = 0.1$, $\mu = 0.5\tau$, the decay rate of the system response is $\delta = 0.2$, and the free tuning parameter is $\alpha = 0.3$. The Procedure 1 yields the PID controller gains:

$$K_P = [32.0294 \;\; 17.1678 \;\; 3.9326],$$
$$K_D = [-62.1339 \;\; -3.0704 \;\; -9.3095],$$
$$K_I = -20.1665.$$

For the analysis of the results, Fig. 2 shows the evolution of the closed-loop system state vector $z(t)$ and its derivative $\dot{z}(t)$ for a unity set-point. In Fig. 3 we can see that the displacement error $e_a(t)$ from the actuated state reaches zero in steady state and the input signal $u(t)$ can also be analyzed.

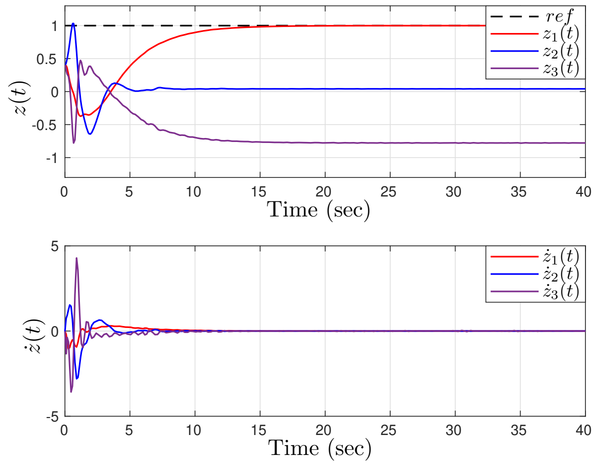

Figura 2. The system closed-loop response. Example 1.

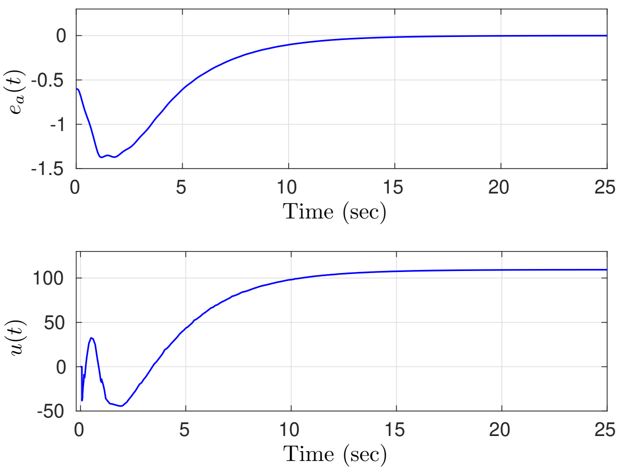

Figura 3. Closed-loop error response on the actuated state vector $e_a(t)$ and control signal $u(t)$. Example 1.

4.2 Example 2

Consider the system studied in Ram et al. (2011) and Araújo (2018), represented by matrices:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad U = B^T. \tag{11}$$

For this example we considered a delay of $\tau = 0.3$, $\mu = 0.5\tau$, with a decay rate equal to $\delta = 0.2$, and the free tuning parameter $\alpha = 0.8$. The PID controller gains obtained through the Procedure 1 are

$$K_P = [-0.3820 \quad -0.3370], \; K_D = [-1.7233 \quad -1.3550],$$
$$K_I = -0.6712.$$

For results analysis purposes, the Fig. 4 shows the response evolution of the closed-loop system state vector $z(t)$ and its derivative $\dot{z}(t)$ to a constant set-point (dashed line) on the actuated degree of freedom. The displacement error $e_a(t)$ of the actuated state and the input signal $u(t)$, can are shown in Fig. 5.

Now for comparison purposes we consider a constant time-delay $\tau = 0.1$ and we design a PID controller applying the proposed method and a controller as proposed in Ram et al. (2011). Fig. 6 presents the system states evolution considering both controllers, where we observe that the actuated state $z_1(t)$ follows the reference only when the proposed PID controller is applied, showing the advantage of the integral action in the control law.

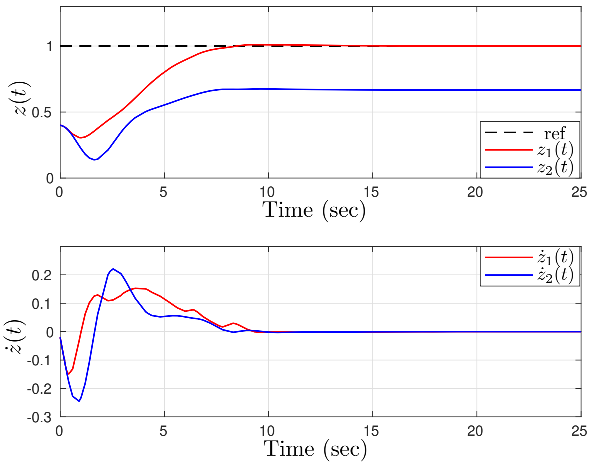

Figura 4. The system closed-loop response. Example 2.

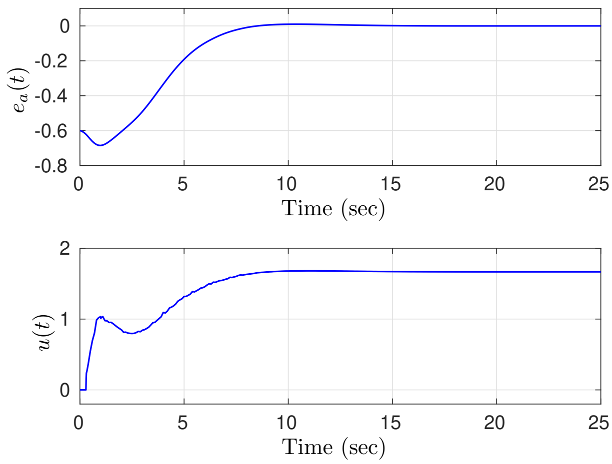

Figura 5. Closed-loop error response on the actuated state vector $e_a(t)$ and control signal $u(t)$. Example 2.

Ram et al. (2009)

4.3 Example 3

Consider the system with more than one independent control force studied in Xia et al. (2019):

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 2.5 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix},$$

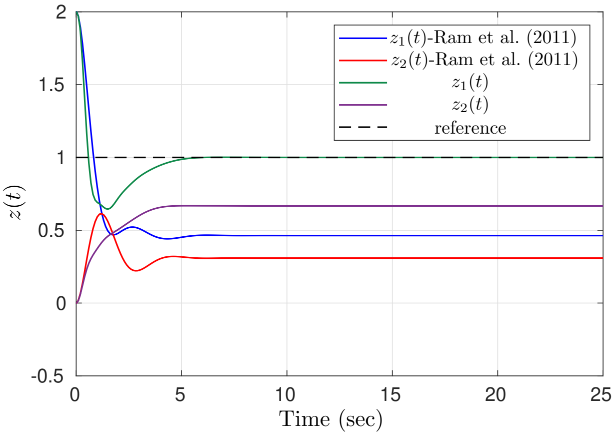

Figura 6. The system closed-loop responses with the proposed controller and the one proposed in Ram et al. (2011). Example 2.

$$K = \begin{bmatrix} 10 & -3 & -4 \\ -3 & 3 & 0 \\ -4 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (12)$$

For this example we considered a delay of $\tau = 0.5$, $\mu = 0.5\tau$, with a decay rate equal to $\delta = 0.1$, and the free tuning parameter $\alpha = 0.8$. The PID controller gains obtained through the Procedure 1 are

$$K_p = \begin{bmatrix} 0.0077 & -0.2843 & 0.5601 \\ 0.0673 & 0.2588 & -0.7424 \end{bmatrix},$$
$$K_d = \begin{bmatrix} -0.2156 & -0.2933 & 0.8126 \\ -0.1286 & -0.1903 & -1.8294 \end{bmatrix},$$
$$K_i = \begin{bmatrix} -1.8664 & 1.6509 \\ 2.3014 & -2.2047 \end{bmatrix}.$$

Fig. 7 shows the evolution of the closed-loop state vector $z(t)$, where the two actuated states converge to the reference, and its derivative $\dot{z}(t)$ for a unitary set-point. Since the decay rate δ considered in this example is very small, the states converge slowly, this parameter can be changed according to the need of the project. The error of the actuated states $e_a(t)$ and the input signal $u(t)$ are shown in Fig. 8. Fig. 9 shows the behavior of the closed-loop step response considering two PID controllers designed by the proposed methodology for $\tau = 0.5$ but one considered $\delta = 0.2$ and the other $\delta = 0.001$. Therefore, as expected one can see the effect of the decay rate parameter, that is, higher values of δ lead to shorter transients.

5. CONCLUSION

A robust framework for proportional-integral-derivative (PID) multivariable design was proposed to control time-varying delay systems modeled by second-order differential equations. The method guarantees a pre-established exponential decay rate for second-order systems subject to time-varying delay, using the Lyapunov-Krasovskii stability theory based on linear matrix inequalities (LMI). Integral action was able to provide additional design flexibility, the design strategy can be applied to obtain constant reference tracking for an actuated subspace of underacted

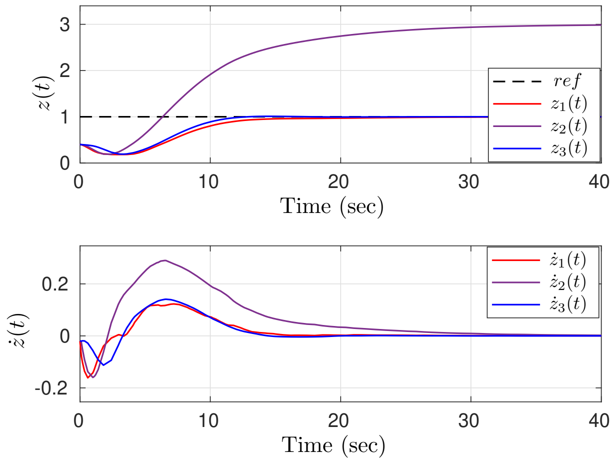

Figura 7. The system closed-loop response. Example 3.

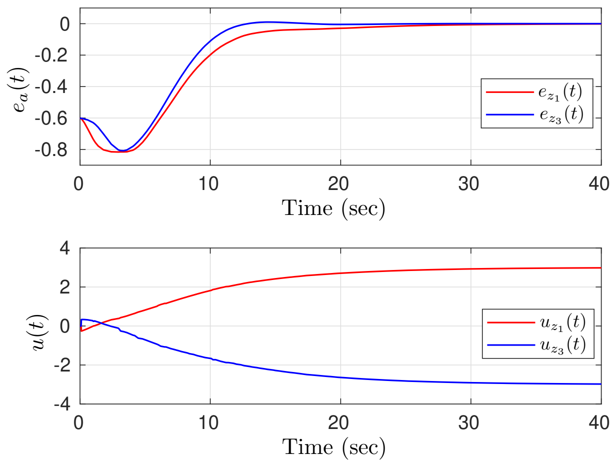

Figura 8. Closed-loop error response on the actuated state vector $e_a(t)$ and control signal $u(t)$. Example 3.

systems. Three numerical examples were presented to demonstrate the effectiveness of the proposed method.

ACKNOWLEDGEMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Codes 001, Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brasil (CNPq: 429819/2018-8 and 312034/2020-2), and Fundação de Amparo à Pesquisa do Estado de Minas Gerais - Brasil (FAPEMIG: APQ-00543-17).

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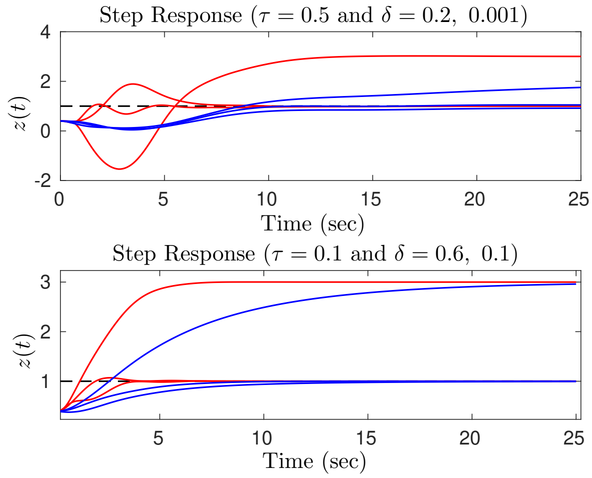

Figura 9. Closed-loop step response for designed controllers with parameter δ. In the first figure: red line ($\delta = 0.2$) and blue line ($\delta = 0.001$), second figure: the red line ($\delta = 0.6$) and blue line ($\delta = 0.1$). Example 3.

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Appendix A. AUXILIARY RESULTS

Lemma 1. (Fenili et al., 2014): Consider the system $\dot{x}(t) = Ax(t) + A_d x(t - d(t))$. Let $\tau > 0$ and $0 \leq \mu \leq \tau$ be given , such that $d(t) \in [\tau - \mu, \tau + \mu]$, and $\delta > 0$, the exponential convergence rate. So the system with $d(t) \in [\tau - \mu, \tau + \mu]$ is exponentially stable, with exponential convergence rate δ, if there are matrices of appropriate dimensions: $F, G, P = P^T, S = S^T, Q, R_1 = R_1^T, R_2, R_3 = R_3^T, Z = Z^T$, such that the LMIs below are satisfied

$$\begin{bmatrix} P & \star \\ Q^T & \varepsilon_1 S \end{bmatrix} > 0, \tag{A.1}$$

where $\varepsilon_1 = e^{-2\delta\tau}/\tau$,

$$R = \begin{bmatrix} R_1 & \star \\ R_2 & R_3 \end{bmatrix} > 0, \tag{A.2}$$

and

$$\left[\begin{array}{c|c} \Xi & \star \\ \hline \Gamma^T & \varepsilon_2^{-1}\mu Z \end{array}\right] < 0, \tag{A.3}$$

where $\varepsilon_2 = e^{-2\delta(\tau+\mu)}$, $\Gamma^T = \mu[A_d^T F^T \quad \alpha A_d^T G^T \quad 0 \quad 0]$ and Ξ is give:

$$\Xi = \begin{bmatrix} \mathcal{F} & \star & \star & \star \\ P + \tau R_2 - \epsilon_2(F^T - GA) & \tau R_3 + 2\mu Z - \epsilon_2(G + G^T) & \star & \star \\ \epsilon_1 R_3^T - Q^T + \epsilon_2 A_d^T F^T & \epsilon_2 A_d^T G^T & -\epsilon_1(R_3 + \tau S) & \star \\ 2\delta Q^T - \epsilon_1 R_2^T & Q^T & \epsilon_1 R_2^T & -\epsilon_1 R_1 \end{bmatrix} \tag{A.4}$$

where $\mathcal{F} = 2\delta P + Q + Q^T + \tau R_1 - \epsilon_1 R_3 + S + \varepsilon_2(AF^T + FA^T)$.