# Rational State-Feedback Control for Discrete-Time State-Polynomial LPV Systems ⋆

**Márcio J. Lacerda** * **Gabriel O. Ferreira** ** **Valter J. S. Leite** ***

* *Control and Modelling Group (GCOM), Department of Electrical Engineering, Federal University of São João del-Rei, São João del-Rei, MG, 36307-352, Brazil. (e-mail: lacerda@ufsj.edu.br)*
** *Consiglio Nazionale delle Ricerche (CNR), Department of Electrical, Electronics, and Communications Engineering, Politecnico di Torino, Torino, Piemonte, 10129, Italy. (e-mail: gabriel.oliveiraferreira@ieiit.cnr.it)*
*** *Department of Mechatronics Engineering, CEFET-MG, Campus Divinópolis, Divinópolis, MG, 36307-352, Brazil. (e-mail: valter@ieee.org)*

**Abstract:** This paper addresses the state-feedback control problem for state-polynomial discrete-time linear parameter varying systems. The sum of squares formulation is employed to write the conditions. Two approaches are presented, the first one makes use of the Lyapunov function to recover the gain matrices, providing state-feedback gains which may be rational in the time-varying parameter, and linear in the state variables. The second formulation allows the design of rational state-feedback control gains concerning both the time-varying parameter and the state variables. Numerical experiments borrowed from the literature are employed to illustrate the efficacy of the proposed method.

**Resumo**: Este artigo aborda o problema de controle por realimentação de estados para sistemas polinomiais nos estados de tempo discreto e sujeitos a parâmetros variantes no tempo. A formulação baseada na técnica de soma de quadrados é empregada para escrever as condições. Duas abordagens são apresentadas, a primeira faz uso da função de Lyapunov para recuperar as matrizes do ganho, fornecendo ganhos por realimentação de estados que podem ser racionais nos parâmetros variantes no tempo e lineares com as variáveis de estado. A segunda formulação permite o projeto de ganhos de controle por realimentação de estado racionais considerando tanto os parâmetros variantes no tempo quando as variáveis de estado. Experimentos numéricos retirados da literatura são usados para ilustrar a eficácia do método proposto.

*Keywords:* State-feedback control; State polynomial systems; Time-varying parameters.

*Palavras-chaves:* Controle por realimentação de estados; Sistemas polinomiais nos estados; Parâmetros variantes no tempo.

## 1. INTRODUCTION

The study of nonlinear systems has experienced a rise in the last decades, mainly due to the several tools to provide stability certificates and control design for this class of systems. One may cite the celebrated T-S (Takagi-Sugeno) fuzzy models (Takagi e Sugeno, 1985) and techniques based on LPV (Linear Parameter Varying), and quasi-LPV systems (Mohammadpour e Scherer, 2012). These techniques conveniently rewrite the systems in terms of simple components, to analyze and design controllers and filters. However, most of the results in the literature are concerned with linear systems, i.e., the state space representation obtained from the T-S fuzzy systems or the LPV representation as a linear function of the states. For instance, in (Cherifi et al., 2019) global stabilization for T-S systems with piecewise continuous membership functions is studied. The approach is based on a Non-Quadratic Lyapunov Function, which considers the mean values of the membership functions over an interval. Concerning the LPV representation, one may see papers dealing with time-varying parameters that belong to a polyhedral set (Geromel e Colaneri, 2006), or more recently, piecewise constant parameters (Briat, 2015) and differentiable parameters (Briat e Mustafa, 2017).

The study of nonlinear state polynomial systems has attracted attention with the development of techniques based on the sum of squares (SOS) (Papachristodoulou et al., 2013). The formulation has been employed to consider stability analysis problems Ahmadi e Parrilo (2011), control design, and also the filtering problem (Li et al., 2012; Lacerda et al., 2015). For instance, in (Jennawasin e Banjerdpongchai, 2018), state-feedback controllers for continuous-time state-polynomial systems with bounded magnitudes of control input are derived. The approach is based on rational Lyapunov functions and

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the convex optimization problem is addressed by employing SOS decomposition. SOS is also used in (Zhao e Wang, 2009), where state-feedback controllers for continuous time-varying state-polynomial systems are derived with parameter and state-dependent Lyapunov functions. In (Ebenbauer e Allgower, 2006), stability analysis and design of state-feedback controllers for the same class of systems are also studied. The authors propose an approach based on dissipation inequalities and the sum of squares decomposition. In (Ferreira et al., 2020), the SOS method is used to design state feedback controllers for continuous-time state-polynomial systems with time-varying parameters, where the controllers depend simultaneously on states and a filtered version of the time-varying parameters. The conditions provide global stabilization certificates. The designed controllers minimize the $\mathcal{L}_2$ gain from the input disturbance to the output of the continuous-time LPV state-polynomial system.

In contrast with the continuous-time state-polynomial systems, the results for discrete-time still are scarce. Moreover, when the state-polynomial system counts with the presence of time-varying parameters this difference is more noticeable. In (Saat et al., 2012), conditions to provide state-feedback controllers for state-polynomial discrete-time systems with polytopic uncertainties are developed. The conditions are based on a parameter-dependent Lyapunov function and an integral action. Global stability certificates are obtained. In (Nasiri et al., 2018), the state-feedback control problem is tackled by considering discrete-time polynomial fuzzy models, the authors propose an approach based on the use of an integrator to guarantee the global stability of the closed-loop systems. The method proposed in (Chen et al., 2014) is also concerned with the stabilizability problem for discrete-time polynomial fuzzy systems. However, only a local stability certificate is provided.

This paper presents new conditions for the design of state-feedback controllers for state-polynomial LPV discrete-time systems. The conditions are written with the use of the sum of squares formulation. The first formulation makes use of the Lyapunov function to recover the state-feedback gain that is linear in the state-variable, and rational in the time-varying parameter. On the other hand, the second formulation proposed in this paper uses a rational gain concerning both the time-varying parameter and the state variables. Numerical experiments are presented to illustrate the effectiveness of our approach.

The remainder of this paper is structured as follows. The preliminaries and the problem formulation are presented in Section 2. The main results are developed in Section 3. Section 4 illustrates the effectiveness of the proposed approach through a numerical experiment, and Section 5 concludes the paper.

**Notation:** $I$ and $0$ denote identity and null matrices of proper dimension, respectively. $\star$ indicates a block induced by symmetry. The transpose of any matrix $X$ is represented by $X^T$, and $\mathbb{R}^{m\times n}$ is the set of real $m \times n$ matrices. If $f(x)$ is SOS, then $f(x) \in \Sigma[x]$.

## 2. PRELIMINARIES

Consider the following state-polynomial LPV system

$$x_{k+1} = A(\alpha_k, x_k)x_k + B(\alpha_k, x_k)u_k, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^{n_u}$ is the control input vector, $k \in \mathbb{N}$ is the time instant. The polynomial LPV matrices in (1) can be generically represented as

$$Z(\alpha_k, x_k) = \sum_{i=1}^{N} \alpha_{k,i} Z_i(x_k), \quad \alpha_k \in \Lambda_N, \tag{2}$$

where $Z_i(x_k)$, $i = 1, \dots, N$, are the vertices of the polytope and $\Lambda_N$ is the unit simplex:

$$\Lambda_N = \left\{ \alpha_k \in \mathbb{R}^N : \sum_{i=1}^{N} \alpha_{k,i} = 1,\ \alpha_{k,i} \geq 0,\ i = 1, \dots, N \right\}.$$

The vertices of the LPV matrices in (1) are described as state-polynomial matrices that may contain monomials of the state variables $x$ up to a certain degree.

The main objective in this paper is to design a control law ensuring the closed-loop stability of (1). To achieve this end, the following polynomial gain scheduling state-feedback control law is considered

$$u_k = K(\alpha_k, x_k), \tag{3}$$

where $K(\alpha_k, x_k) \in \mathbb{R}^{n_u \times n}$ is a polynomial LPV matrix with the same structure presented in (2). Furthermore, the time-varying parameter $\alpha_k$ is considered to be available, measured, or estimated online. Taking into account the state-feedback controller (3), the closed-loop system reads

$$x_{k+1} = \tilde{A}(\alpha_k, x_k)x_k, \tag{4}$$

with $\tilde{A}(\alpha_k, x_k) = A(\alpha_k, x_k) + B(\alpha_k, x_k)K(\alpha_k, x_k)$.

The conditions proposed in this paper are obtained via Lyapunov Theory, and the matrices constraints certificates are secured through the sum of squares (SOS) decomposition. A multiple variables polynomial $F(x_1, x_2, \dots, x_n)$ of degree $2d$ is SOS, if it can be written according to

$$F(x_1, x_2, \dots, x_n) = \sum_{i=1}^{m} f_i^2(x_1, x_2, \dots, x_n), \tag{5}$$

where each polynomial $f_i(x_1, x_2, \dots, x_n)$ has degree lower or equal to $d$. Equation (5) is clearly semi-positive definite and can be written as

$$F(x) = z^T Q z, \tag{6}$$

where $z$ is a vector containing monomials of degree up to $d$ of $(x_1, x_2, \dots, x_n)$. $Q \geq 0$ and can be decomposed, for instance by using the Cholesky factorization, as $Q = V^T V$. Subsequently, one may calculate a vector $f$ containing all $f_i$'s according to

$$f(x_1, x_2, \dots, x_n) = Vz. \tag{7}$$

In the sequel, we will establish conditions to design state-feedback controllers which can stabilize the closed-loop system (4).

## 3. MAIN RESULTS

**Theorem 1.** If there exist parameter dependent matrices $P(\alpha_k)$ and polynomial parameter dependent matrices $Z(\alpha_k, x_k)$ such that

$$M_{i,z} - \epsilon I \in \Sigma[x],$$

$i = 1,\ldots,N$, $z = 1,\ldots,N$, with

$$M_{i,z} = \begin{bmatrix} P_i & Z_i(x_k)^T B_i(x_k)^T + P_i A_i(x_k)^T \\ \star & P_z \end{bmatrix}, \quad (8)$$

and

$$Q_{i,j,z} - \epsilon I \in \Sigma[x],$$

$i = 1,\ldots,N-1$, $j = i+1,\ldots,N$, $z = 1,\ldots,N$, with

$$Q_{i,j,z} = \begin{bmatrix} P_i + P_j & \Phi_{ij} \\ \star & 2P_z \end{bmatrix}, \quad (9)$$

where

$$\Phi_{ij} = Z_j(x_k)^T B_i(x_k)^T + P_j A_i(x_k)^T + Z_i(x_k)^T B_j(x_k)^T + P_i A_j(x_k)^T, \quad (10)$$

then, the discrete-time state-polynomial LPV system (4) is asymptotically stable and the polynomial LPV state-feedback controller is given by

$$K(\alpha_k,x_k) = Z(\alpha_k,x_k)P(\alpha_k)^{-1}.$$

**Proof.** Note that if Theorem 1 is satisfied, then (8) and (9) are positive definite matrices. Multiplying (8) by $\alpha_{k,i}^2$, $i = 1,\ldots,N$, multiplying (9) by $\alpha_{k,i}\alpha_{k,j}$, $i = 1,\ldots,N-1$, $j = i+1,\ldots,N$, and summing up one has

$$\begin{bmatrix} P(\alpha_k) & Z(\alpha_k,x_k)^T B(\alpha_k,x_k)^T + P(\alpha_k)A(\alpha_k,x_k)^T \\ \star & P_z \end{bmatrix} > 0.$$

Multiplying the last matrix by $\beta_{k,z}$, $z = 1,\ldots,N$, and summing up yields

$$\begin{bmatrix} P(\alpha_k) & Z(\alpha_k,x_k)^T B(\alpha_k,x_k)^T + P(\alpha_k)A(\alpha_k,x_k)^T \\ \star & P(\beta_k) \end{bmatrix} > 0. \quad (11)$$

By replacing $Z(\alpha_k,x) = K(\alpha_k,x_k)P(\alpha_k)$, and applying a congruence transformation in (11) with

$$\begin{bmatrix} P(\alpha_k)^{-1} & 0 \\ 0 & P(\beta_k)^{-1} \end{bmatrix},$$

one can write

$$\begin{bmatrix} P(\alpha_k)^{-1} & \tilde{A}(\alpha_k,x_k)^T P(\beta_k)^{-1} \\ \star & P(\beta_k)^{-1} \end{bmatrix} > 0. \quad (12)$$

Replacing $P(\alpha_k)^{-1}$ and $P(\beta_k)^{-1}$ by $W(\alpha_k)$ and $W(\beta_k)$ respectively, and applying a Schur complement yields

$$W(\alpha_k) - \tilde{A}(\alpha_k,x_k)^T W(\beta_k)\tilde{A}(\alpha_k,x_k) > 0, \quad (13)$$

with $\beta_k = \alpha_{k+1}$. Therefore, by calling $V(x_k) = x_k^T W(\alpha_k)x_k$, and noting that $W(\alpha_k) > 0$, we have that there exist positive scalars $c_2 > c_1 > 0$ such that

$$c_1\|x\|^2 \le V(x_k) \le c_2\|x\|^2.$$

A possible choice for $c_1$ and $c_2$ is $c_1 = \min_{\alpha_k}(\lambda(W(\alpha_k)))$ and $c_2 = \max_{\alpha_k}(\lambda(W(\alpha_k)))$. By pre- and post-multiplying (13) by $x^T$ and its transpose, respectively, and using the closed-loop equation in (4), we get $V(x_{k+1}) - V(x_k) < 0$. Therefore, there always exists a small enough scalar $c_3 > 0$ such ensuring that $V(x_{k+1}) - V(x_k) < -c_3\|x\|^2$.

The main drawback with the conditions proposed in Theorem 1 is the fact that the state-feedback controller is recovered from the Lyapunov matrix that does not depend on the state vector $x_k$. The gain stills a polynomial function of the state vector $x_k$, once the matrix $Z(\alpha_k,x_k)$ is also employed to recover it. The next result presents a condition that does not employ the Lyapunov matrix to recover the controller and allows the design of rational controllers in both $\alpha_k$ and $x_k$.

**Theorem 2.** If there exist parameter dependent matrices $P(\alpha_k)$ and polynomial parameter dependent matrices $X(\alpha_k,x_k)$ and $Z(\alpha_k,x_k)$ such that

$$\Psi_{i,z} - \epsilon I \in \Sigma[x],$$

$i = 1,\ldots,N$, $z = 1,\ldots,N$, with

$$\Psi_{i,z} = \begin{bmatrix} X_i(x_k) + X_i(x_k)^T - P_i & \star \\ A_i(x_k)X_i(x_k) + B_i(x_k)Z_i(x_k) & P_z \end{bmatrix}, \quad (14)$$

and

$$\Theta_{i,j,z} - \epsilon I \in \Sigma[x],$$

$i = 1,\ldots,N-1$, $j = i+1,\ldots,N$, $z = 1,\ldots,N$, with

$$\Theta_{i,j,z} = \begin{bmatrix} \mathcal{R} & \star \\ \mathcal{S} & 2P_z \end{bmatrix}, \quad (15)$$

where

$$\mathcal{R} = X_i(x_k) + X_j(x_k) + X_i(x_k)^T + X_j(x_k)^T - P_i - P_j,$$

$$\mathcal{S} = A_i(x_k)X_j(x_k) + B_i(x_k)Z_j(x_k) + A_j(x_k)X_i(x_k) + B_j(x_k)Z_i(x_k),$$

then, the discrete-time state-polynomial LPV system (4) is asymptotically stable and the polynomial LPV state-feedback controller is given by

$$K(\alpha_k,x_k) = Z(\alpha_k,x_k)X(\alpha_k,x_k)^{-1}.$$

**Proof.** If Theorem 2 is satisfied, then following the same steps used in Theorem 1 one can write

$$\begin{bmatrix} X(\alpha_k,x_k) + X(\alpha_k,x_k)^T - P(\alpha_k) & \star \\ A(\alpha_k,x_k)X(\alpha_k,x_k) + B(\alpha_k,x_k)Z(\alpha_k,x_k) & P(\beta_k) \end{bmatrix} > 0. \quad (16)$$

Replacing $Z(\alpha_k,x_k) = K(\alpha_k,x_k)X(\alpha_k,x_k)$ yields

$$\begin{bmatrix} X(\alpha_k,x_k) + X(\alpha_k,x_k)^T - P(\alpha_k) & \star \\ \tilde{A}(\alpha_k,x_k)X(\alpha_k,x_k) & P(\beta_k) \end{bmatrix} > 0. \quad (17)$$

By exploiting the inequality

$$X(\alpha_k,x_k)^T P(\alpha_k)^{-1}X(\alpha_k,x_k) \ge X(\alpha_k,x_k) + X(\alpha_k,x_k)^T - P(\alpha_k),$$

one gets

$$\begin{bmatrix} X(\alpha_k,x_k)^T P(\alpha_k)^{-1}X(\alpha_k,x_k) & \star \\ \tilde{A}(\alpha_k,x_k)X(\alpha_k,x_k) & P(\beta_k) \end{bmatrix} > 0. \quad (18)$$

Pre- and post-multiplying (18) by $S$ and $S^T$ respectively, with

$$S = \begin{bmatrix} X(\alpha_k,x_k)^{-T} & 0 \\ 0 & P(\beta_k)^{-1} \end{bmatrix},$$

one has exactly (12). Then, the same steps performed in Theorem 1 can be followed to conclude the proof.

**Remark 3.** The $\epsilon$ value employed in the conditions, is a small positive definite function used to certify that the conditions are strictly positive. The choice for such a parameter is not unique, and it could be also state-dependent. For instance,

$$\epsilon = \sum_{k=1}^{n} \epsilon_k x_k^2,$$

where $\epsilon_k$, $k = 1,\ldots,n$, are positive scalar variables. The reader is referred to (Papachristodoulou e Prajna, 2002, 2005) for a more detailed discussion.

**Remark 4.** Note that, differently from Theorem 1, the LPV state-feedback controller obtained with Theorem 2 may be rational in both parameters $\alpha_k$ and state $x_k$.

**Corollary 5.** If there exist symmetric parameter-dependent matrices $P(\alpha_k)$ and matrices $X$ and $Z$ such that

$$\Psi_{i,z} - \epsilon I \in \Sigma[x],$$

$i = 1,\ldots,N$, $z = 1,\ldots,N$, with

$$\Psi_{i,z} = \begin{bmatrix} X + X^T - P_i & \star \\ A_i(x_k)X + B_i(x_k)Z & P_z \end{bmatrix}, \quad (19)$$

then, the polynomial LPV system (4) is asymptotically stable and the robust state-feedback control gain is given by

$$K = ZX^{-1}.$$

**Proof.** The proof follows the same steps presented in the Proof of Theorem 2 by using $Z = KX$.

**Remark 6.** Note that Corollary 5 presents a condition where constant matrices $X$ and $Z$ are employed. In this case, a robust controller is obtained. However, the Lyapunov function employed is a parameter-dependent Lyapunov function.

## 4. NUMERICAL EXPERIMENTS

To illustrate the potential of the proposed method some numerical experiments are considered. The routines were implemented in MATLAB R2014a using the SOSTOOLS (Papachristodoulou et al., 2013) and the solver SeDuMi (Sturm, 1999).

Consider the polynomial LPV system (1) with matrices

$$A_1(x_k) = \begin{bmatrix} -1 & 0.1x_{2_k} \\ 0.2 & 1 \end{bmatrix}, \; A_2(x_k) = \begin{bmatrix} 1 & 0.1x_{2_k} \\ 0.2 & 1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

This system was borrowed from (Nasiri et al., 2018), in the context of polynomial fuzzy systems. In that paper, an integrator has been employed, and a convex solution in terms of SOS was provided. The controller obtained for this example involved polynomial matrices with degrees 7 and 8. The global stability was certified and a polynomial Lyapunov function radially unbounded has been used to certify the closed-loop stability.

The method proposed in (Chen et al., 2014) was also able to provide a stabilizing controller, by using polynomials with degrees 6 and 8 in the state-feedback gains, however, the Lyapunov function employed only provided a local stability certificate was shown in (Nasiri et al., 2018).

Theorem 1 with a constant matrix $P$ and $\epsilon = 10^{-3}$ is able to provide a state-feedback controller that stabilizes the system. The matrix $Z(\alpha_k,x_k)$ has been considered only with monomials of the state variable $x_{2_k}$ up the degree two. In this way, the state-feedback controller has the following form

$$K_1(x_k) = [K_{11} \;\; K_{12}], \; K_2(x_k) = [K_{21} \;\; K_{22}], \quad (20)$$

with

$$K_{11} = 0.8456 - 6.6535 \times 10^{-6}x_{2_k} - 5.3009 \times 10^{-11}x_{2_k}^2,$$
$$K_{12} = -0.5357 - 0.1000x_{2_k} + 4.8633 \times 10^{-12}x_{2_k}^2,$$
$$K_{21} = -1.0302 - 6.3553 \times 10^{-6}x_{2_k} + 5.3963 \times 10^{-11}x_{2_k}^2,$$
$$K_{22} = -0.5159 - 0.1000x_{2_k} - 1.4127 \times 10^{-11}x_{2_k}^2.$$

Moreover, the matrix $P$ obtained from the solution is

$$P = \begin{bmatrix} 0.8782 & -0.3522 \\ -0.3522 & 0.6506 \end{bmatrix}.$$

The Lyapunov function is given by $V(x_k) = x_k^T P^{-1} x_k$. Level sets of the Lyapunov function are presented in Figure 1. It can be seen that the Lyapunov function is radially unbounded and it is able to certify the global stability of the closed-loop system.

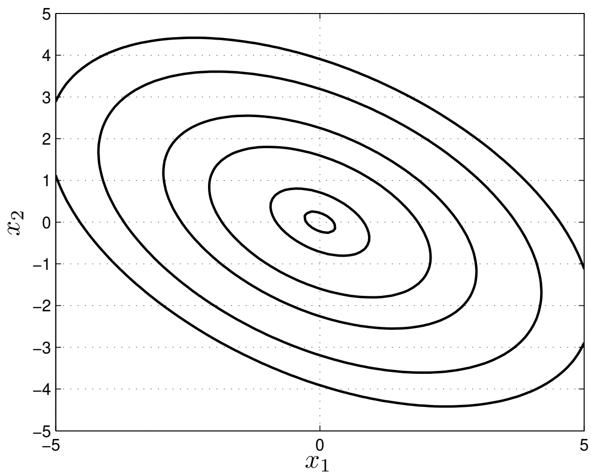


Figure 1. Level sets of the Lyapunov function $V(x_k)$, $\{x_k,\, x_k^T P^{-1} x_k = \xi\}$ for $\xi = [0.1,1,5,10,20,30]$.

To evaluate the performance of the controller the following rule has been considered for the LPV parameters. This is the same rule considered in (Nasiri et al., 2018).

$$\alpha_{k,1} = \frac{1}{2}\left(1 + \sin(x_{1_k})\right), \quad \alpha_{k,2} = 1 - \alpha_{k,1}.$$

Figure 2 depicts the trajectories for different initial conditions. In this case, 100 randomly generated initial conditions have been considered. It can be seen that the trajectories always converge to the origin. Each trajectory

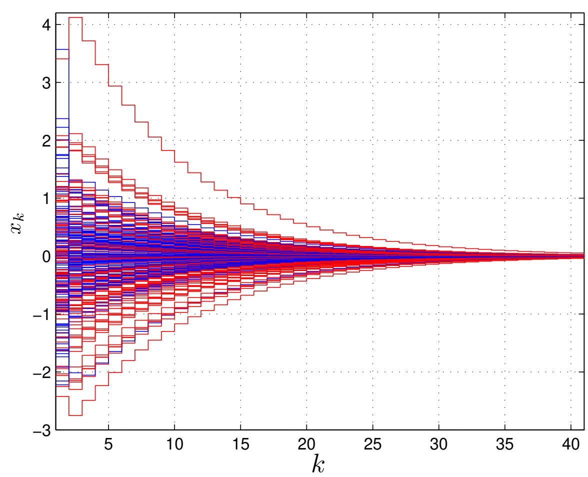


Figure 2. State trajectories for 100 randomly generated initial conditions with the state-feedback control law given in (20).

presented in Figure 2 is associated with a Lyapunov function that is positive definite and monotonically decreasing along the trajectories. For instance, considering an initial condition $[-1\ 1]^T$, the trajectory of the Lyapunov function is shown in Figure 3.

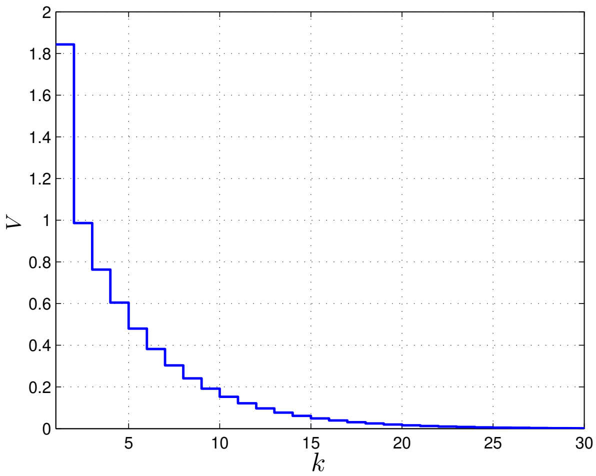


Figure 3. Trajectory of the Lyapunov function along the trajectories of the system.

Theorem 2 is also able to stabilize this system. For instance, by considering a constant matrix $P$, a matrix $X(x_{2_k})$ of degree up to one in $x_{2_k}$ and matrices $Z(\alpha_k,x_{2_k})$ with degree 1 in $\alpha$ and degree up to one in $x_{2_k}$, a feasible solution can be found. In this case, a parameter-dependent rational feedback gain in $x_k$ is obtained, since the control is obtained by employing the inverse of the matrix $X(x_{2_k})$. In this Example, Corollary 5 fails to find a feasible solution.

To further test the robustness of the proposed conditions consider the matrices $A_1(x_k)$, and $A_2(x_k) = \rho A_2(x_k)$. The goal is to obtain the maximum value of $\rho$ such that the conditions are feasible. Theorem 1 achieved $\rho = 83.42$, while Theorem 2 attained $\rho = 400$. This shows that Theorem 2 can provide controllers for a wide range of variations of the vertices of the LPV state-polynomial system. This corroborates the fact that the use of a parameter-dependent rational state-feedback controller may provide less conservative results than an LPV controller.

## 5. CONCLUSIONS

This work presented new conditions to design state-feedback controllers for discrete-time state-polynomial LPV systems. The control gains present a rational structure in both the time-varying parameter and the state variables. Two formulations are presented, the first based on the use of the Lyapunov matrix to recover the control gain, and the second makes use of a slack variable to design the controllers. Different from existing methods, the proposed Lyapunov function does not depend on the state variables, producing global stability certificates for the closed-loop systems. Numerical experiments with an example borrowed from the literature illustrated the efficacy of our approach. In future research, the proposed conditions will be extended to take into account constraints in the input signal, and the use of the $\ell_2$ gain to consider the presence of input disturbances.

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