$\mathcal{H}_\infty$ DYNAMIC OUTPUT FEEDBACK CONTROLLER DESIGN BY REFERENCE MODEL AND TWO DEGREES OF FREEDOM CONTROL: AN LMI APPROACH

Ariany C. de Oliveira [*,**] **Víctor C. S. Campos** [***]
Leonardo A. Mozelli [***]

[*] *Graduate Program in Electrical Enginnering, Universidade Federal de Minas Gerais (UFMG), Belo Horizonte, MG, Brasil*
[**] *Department of Electrical - CEFET, Nepomuceno, MG, Brasil*
[***] *Department of Electronics Engineering - UFMG*

Abstract: In this article, the $\mathcal{H}_\infty$ model reference problem in two degrees of freedom control methodology using dynamic output feedback for continuous-time linear systems is presented. The controller design is formulated in terms of Linear Matrix Inequalities (LMIs). An illustrative example is presented to demonstrate the utility of the proposed synthesis procedure.

Resumo: Neste artigo é apresentado o problema de controle $\mathcal{H}_\infty$ por modelo de referência utilizando uma metodologia de controle de dois graus de liberdade (2DOF) . O projeto do controlador é formulado em termos de desigualdades matriciais lineares (LMIs). Um exemplo numérico é apresentado para ilustrar a utilidade do procedimento de síntese proposto.

Keywords: Dynamic output feedback control; reference model; guaranteed cost; linear matrix inequalities; integral control; 2DOF methodology.

Palavras-chaves: Realimentação dinâmica de saída; modelo de referência; custo garantido; desigualdades matriciais lineares; controle integral; metodologia 2DOF.

1. INTRODUCTION

Control systems often required a balance between two important objectives: reference tracking and disturbance rejection. In this case, the design of the control system is a multiobjective problem. The simplest strategy, one degree of freedom (1DOF) control systems results in hard or even unfeasible optimization problems concerning these trade-offs (Limebeer et al., 1993). One different result can be achieved with two degrees of freedom (2DOF) control systems, which combine the feedforward controller and feedback controller to achieve the desired tracking performance (Peng et al., 2016). In this way, there is the possibility to adjust tracking response and disturbance independently (Araki and Taguchi, 2003; Peng et al., 2016).

Therefore, 2DOF control systems are a matter of interest for many researchers. Some works use $\mathcal{H}_\infty$ to minimize the error between model and system. In Gören (2003) is proposed a condition based on Linear Matrix Inequality (LMI) to solve the $\mathcal{H}_\infty$ model matching problem (MMP) in two degrees of freedom control structure for continuous time system. The results are based on the concept of the standard $\mathcal{H}_\infty$ Optimal Control Problem (OCP) formulated in Gahinet and Apkarian (1994) and Gören and Murat (2002). As a novelty, they use an output feedback configuration in the formulation of the problem. In Gören (2003), previous work is transferred to discrete-time context and leads to qualitatively similar results.

Model reference control is a problem of theoretical and practical importance (Kučera, 2015). The principal characteristic of this approach is that response specifications are given by a reference model with all the desired characteristics for the controlled system (Souza et al., 2021). The mathematical formulation of the problem is attractive and has enticed great interest.

Several approaches have been developed to deal with this topic in several contexts. For instance, a review of this problem solved by state feedback can be seen in Kučera and Toledo (2014). In the robust control context, Bachur et al. (2010) developed a model reference control based on LMIs for uncertain linear continuous-time systems based on a two-step iterative procedure. In Bachur et al. (2011), a reference model control for uncertain linear discrete-time systems was developed, also based on a two-step iterative procedure. In Souza et al. (2021) a $\mathcal{H}_\infty$ reference model problem is solved for an uncertain linear continuous-time system, in this case, an iterative procedure was used to solve conditions formulated in terms of LMIs. In the three previous cases, a dynamic output feedback controller was used. Reference model to Takagi-Sugeno fuzzy systems have been considered in Campos et al. (2011); Tseng et al. (2001); Andrea et al. (2008); Mansouri et al. (2009). When system parameters are unknown and/or change with time, adaptive control is a technique used for providing a desired level of performance through real-time estimation of parameters (Shekhar and Sharma, 2018). The Model Reference Adaptive Control (MRAC) has been used to solve Adaptive Control problems in real applications, as we see in Shekhar and Sharma (2018); Kaufman et al. (2012); Cao (2008); Yin and Lee (1995).

Given the context presented, connecting the problem of model reference tracking and disturbance rejection is of practical importance since the objective is tracking model reference when the system is affected by external disturbances (Gören and Çamlibel, 1997). This problem can be solved using a 2DOF control structure (Gören and Çamlibel, 1997).

In this research, based on the techniques developed by Kaufman et al. (2012), we study the problem of reference model control for a class of linear continuous-time systems in two degrees of freedom (2DOF) control structure. The system is controlled by dynamic output feedback including $\mathcal{H}_\infty$ performance and integral action to guarantee disturbance rejection plus asymptotic tracking of the reference model output. The conditions are formulated in terms of LMIs, which are solved with a two-step procedure using the results given by Dullerud and Paganini (2000). An example is used to illustrate the results.

The text is organized as follows. Section 2 brings the problem statement and preliminaries concepts. The $\mathcal{H}_\infty$ design conditions for the dynamic output controller are depicted in Section 3 and a numerical example, in Section 4, illustrates the usefulness of the proposed approach.

Notation: Symmetric positive definite (negative definite) matrices $M = M^T$ are denoted by $M > 0$ ($M < 0$). M^T denotes the transpose of matrix M. The $*$ symbol represents blocks symmetric in a square matrix.

2. PRELIMINARIES

Consider the following continuous-time linear system:

$$\mathcal{G} = \begin{cases} \dot{x}(t) = Ax(t) + B_2u(t) + B_1w(t), \\ z(t) = C_1x(t) + D_{12}u(t) + D_{11}w(t), \\ y(t) = C_2x(t) + D_{21}w(t), \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the control input signal, $w(t) \in \mathcal{R}^v$ is the disturbance, $z(t) \in \mathcal{R}^q$ is the controlled output and $y(t) \in \mathcal{R}^q$ is the measured output. The system's dynamics are described by matrices A, B_2, B_1, C_1, D_{12}, D_{11}, D_{21}, C_2 and D_{21}, which are required to have appropriate dimensions.

This linear controlled system, $\mathcal{G}$, with the presence of unknown bounded disturbances, $w(t)$, must follow the output of the reference model:

$$\mathcal{G}_m = \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B_2\hat{u}(t), \\ \hat{y}(t) = C_2\hat{x}(t). \end{cases}, \tag{2}$$

where $\hat{x}(t) \in \mathcal{R}^n$, $\hat{u}(t) \in \mathcal{R}^m$ and $\hat{y}(t) \in \mathcal{R}^q$ represent the states, control input and measured output of the reference model, respectively. In that regard, the control system's objective is to drive the error signal, $e_y(t) = y - \hat{y}$, between system (1) and model (2) to zero. To attain this goal, the control input signal is given by

$$u(t) = \hat{u}(t) + s(t), \tag{3}$$

the signal $r(t)$ represents the desired tracking trajectory given by state feedback (of the reference model) of the form

$$\hat{u}(t) = \hat{K}_r r(t) - \hat{K}\hat{x}(t), \tag{4}$$

with $\hat{K}_r$ a reference gain and $\hat{K}$ a state feedback gain, and $s(t)$ given by a dynamic output feedback controller of the form

$$\mathcal{G}_c = \begin{cases} \dot{x}_c(t) = A_kx_c(t) + B_ke_y(t), \\ s(t) = C_kx_c(t) + D_ke_y(t), \end{cases} \tag{5}$$

with $x_c(t) \in \mathcal{R}^{n_k}$ and matrices $A_k \in \mathcal{R}^{n_k \times n_k}$, $B_k \in \mathcal{R}^{n_k \times m_k}$, $C_k \in \mathcal{R}^{p_k \times n_k}$ and $D_k \in \mathcal{R}^{p_k \times m_k}$.

The aim of this paper is to design the controllers in (4) and (5) to address trajectory tracking and disturbance rejection. Controller (4) is designed by pole placement so that the reference model follows a reference with a desired dynamic and zero steady-state error. Controller (5) is designed to minimize the $\mathcal{H}_\infty$ norm between the exogenous input $w(t)$ and output error $e_y(t)$, such that the effect of the disturbance in the system is attenuated. It is important to highlight that by the design methodology proposed, the two problems are solved independently, as described below.

3. TWO DEGREES OF FREEDOM CONTROL METHODOLOGY

It is usual to expect a feedback control system to have a desirable closed-loop dynamic and steady-state response. Ideally, six features are expected: stability, good disturbance rejection ability, good tracking ability, zero steady-state errors, no excessive control action, and lastly, robustness.

However, we know that, in practical situations, it is impossible to satisfy all performance specifications at the same time, and, hence, sometimes it is necessary to balance conflicting criteria (Seborg et al., 2016). One important trade-off example occurs in the design of standard PID controllers, where we have to balance between tracking and disturbance rejection. To solve this conflict, we propose the use of two degrees of freedom design methodology based on a decoupling scheme, as shown in Figure 1.

At the bottom of Figure 1, we have the structure responsible for the set-point tracking. This is formed by a reference model, $\mathcal{G}_m$, controlled with (4) such that it has the desired closed-loop performance, like overshoot and settling time. These can be determined from the eigenvalues of $A - B\hat{K}$, wherein the reference input, $r(t)$, is weighted by $\hat{K}_r$ to guarantee zero steady-state error (which is feasible since the model is perfectly known). The reference model control signal, $\hat{u}(t)$, composes $u(t)$, applied in the system.

For disturbance rejection, as not all states are available to the controller, we employ a dynamic output feedback controller, $\mathcal{G}_c$, to minimize the norm between the disturbance input $w(t)$ and output error $e_y(t)$, given by the difference between system, $\mathcal{G}$ and reference model output. Therefore, if the error is zero, we have perfect tracking and the control applied in the system is the same one used in the reference model. If we have a disturbance, the dynamic output controller attenuates this effect on the error. Lyapunov stability theory can be applied to study the stability of the error between the system and the reference model outputs. Thus, the main contribution of this paper is the proposition of a design methodology

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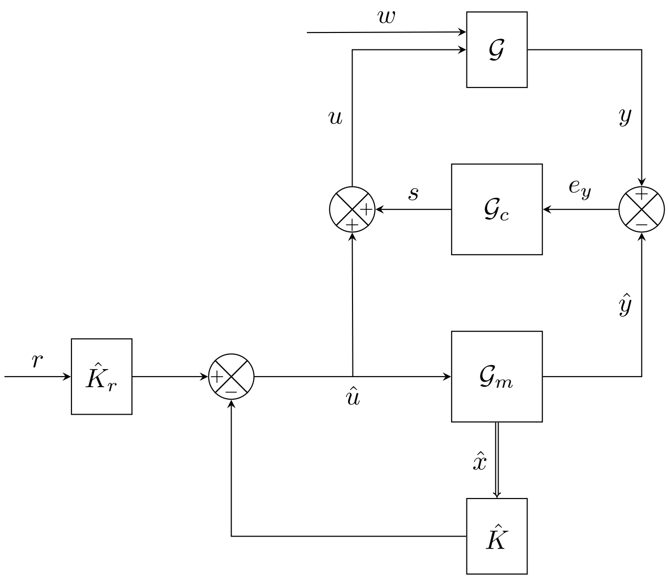

Figure 1. Reference model control with dynamic output feedback. The reference model performance is governed by a full state feedback control, using the gain $\hat{K}$. The plant, $\mathcal{G}$, under the action of disturbances signals, w, must follow the output of the reference model, $\mathcal{G}_m$. This task is accomplished by the second degree of freedom, provided by the dynamic output feedback controller, $\mathcal{G}_c$

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with two degrees of freedom, in that the tracking problem is solved by pole placement and the disturbance rejection problem is solved with LMI conditions that use the $\mathcal{H}_\infty$ norm as a performance criterion.

In order to design the $\mathcal{H}_\infty$ output feedback controller, we slightly adapt the LMI conditions in (Dullerud and Paganini, 2000, chapter 7), which are synthesized below for completion purposes.

A dynamic output feedback controller is designed such that the corresponding closed-loop system is asymptotically stable with a guaranteed $\mathcal{H}_\infty$ norm of γ for the gain between the disturbance $w(t)$ and the controlled output $z(t)$. The closed-loop system can be written as

$$\mathcal{G}_{cl} = \begin{cases} \dot{x}_{cl}(t) = A_{cl}x_{xl}(t) + B_{1_{cl}}w(t), \\ z(t) = C_{cl}x_{cl}(t) + D_{cl}w(t), \end{cases} \tag{6}$$

with $x_{cl} = \left[x^T \ x_c^T\right]^T$ and

$$\begin{aligned} A_{cl} &= \bar{A} + \underline{B}J\underline{C}, \\ C_{cl} &= \bar{C} + \underline{D}_{12}J\underline{C}, \\ B_{cl} &= \bar{B} + \underline{B}J\underline{D}_{21}, \\ D_{cl} &= D_{11} + \underline{D}_{12}J\underline{D}_{21}, \end{aligned}$$

with

$$J = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}, \tag{7}$$

employed as a compact representation of the dynamic output feedback and

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \bar{C} = [C_1 \ 0], \underline{B} = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix},$$

$$\underline{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix}, \bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \underline{C} = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix}, \underline{D}_{12} = [0 \ D_{12}].$$

With that in mind, the following results are employed in the remainder of this work.

Corollary *1.* (Dullerud and Paganini (2000)). Suppose $\hat{G}_{cl}(s) = C_{cl}(sI - A_{cl})^{-1}B_{cl} + D_{cl}$. Then, the following are equivalent conditions.

i Matrix A_{cl} is Hurwitz and $||\hat{G}_{cl}(s)||_\infty < \gamma$.
ii There exists a symmetric positive definite matrix X_L such that

$$\begin{bmatrix} A_{cl}^T X_L + X_L A_{cl} & X_L B_{cl} & C_{cl}^T \\ * & -\gamma I & D_{cl}^T \\ * & * & -\gamma I \end{bmatrix} < 0,$$

or

$$H_{X_L} + Q^T J^T P_{X_L} + P_{X_L}^T JQ < 0, \tag{8}$$

with

$$\begin{aligned} P_{X_L} &= \left[\underline{B}^T X_L \ 0 \ \underline{D}_{12}^T\right], \\ Q &= [\underline{C} \ \underline{D}_{21} \ 0], \\ H_{X_L} &= \begin{bmatrix} \bar{A}^T X_L + X_L \bar{A} & X_L \bar{B}_L & \bar{C}^T \\ * & -\gamma I & D_{11}^T \\ * & * & -\gamma I \end{bmatrix}. \end{aligned}$$

Proof. See Dullerud and Paganini (2000) for the proof.

Theorem *2.* (Dullerud and Paganini (2000)). A synthesis exists for the $\mathcal{H}_\infty$ problem, if and only if there exist symmetric matrices $X > 0$ and $Y > 0$ such that:

$$\begin{bmatrix} N_x & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A^T X + XA & XB_1 & C_1^T \\ * & -\gamma I & D_{11}^T \\ * & * & -\gamma I \end{bmatrix} \begin{bmatrix} N_x & 0 \\ 0 & I \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} N_y & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} AY + YA^T & YC_1^T & B_1 \\ * & -\gamma I & D_{11} \\ * & * & -\gamma I \end{bmatrix} \begin{bmatrix} N_y & 0 \\ 0 & I \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0, \tag{11}$$

with N_x and N_y full-rank matrices whose images satisfy

$$\begin{aligned} Im(N_x) &= ker\,[C_2 \ D_{21}], \\ Im(N_y) &= ker\left[B_2^T \ D_{12}^T\right]. \end{aligned}$$

Proof. See Dullerud and Paganini (2000) for the proof.

Theorem 2 provides conditions to solve the $\mathcal{H}_\infty$ problem, but a reconstruction procedure is necessary for the controller, which can be given by

- **Step 1:** Find a solution X and Y to Theorem 2. Then, matrix $X_L \in \mathcal{R}^{n \times n_k}$ is given by
$$X_L = \begin{bmatrix} X & X_2^T \\ * & I \end{bmatrix},$$
and
$$X - Y^{-1} = X_2 X_2^T.$$
- **Step 2:** Find a solution J, corresponding to the dynamic output feedback controller parameters, by solving (8).

4. MAIN RESULTS

In this section, the main results are presented. Firstly, an optimal $\mathcal{H}_\infty$ controller to solve the proposed control problem is presented. Since optimal $\mathcal{H}_\infty$ controller synthesis has a tendency to produce large controller gains, an auxiliary method is proposed, in which a sub-optimal value for γ is defined and an integral control action is provided to compensate for steady-state errors.

4.1 $\mathcal{H}_\infty$ Dynamic output feedback controller design

In Figure 1, the error between the plant states and the model reference states is

$$e(t) = x(t) - \hat{x}(t). \tag{12}$$

Then, the error dynamics are given by

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t). \tag{13}$$

By making use of (1) and (2) in (13), we have that

$$\dot{e} = Ae(t) + B_2(u - \hat{u}) + B_1 w(t). \tag{14}$$

Since $u(t)$ is given by (3),

$$\dot{e} = Ae(t) + B_2 s(t) + B_1 w(t), \tag{15}$$

which can be employed for our design methodology.

Theorem 3. Consider the continuous-time linear system (1) and the reference model (2). If there exist matrices $X > 0$, $Y > 0$ that satisfy the LMI conditions (9)-(11) for a given γ, an $X_L > 0$ matrix can be reconstructed such that there exist matrices J that satisfy (8) and controller

$$s(t) = C_k x_c(t) + D_k e_y(t), \tag{16}$$

asymptotically stabilizes (15) with an $\mathcal{H}_\infty$ guaranteed cost γ.

Proof. The proof is similar to that of Corollary 1 and Theorem 2 and is omitted.

4.2 $\mathcal{H}_\infty$ dynamic output feedback controller design with integral control

The problem presented earlier can be modified to include an integral control action. In order to do so, in Figure 1 an integrator is added to the forward path of the output error feedback system and is considered as a state of an augmented error system dynamics. The derivative of this integral error, e_i, is given by:

$$\dot{e}_i(t) = C_2 e_y(t).$$

Let the augmented error state space description of (15) be given by

$$\begin{aligned}
\begin{bmatrix} \dot{e}(t) \\ \dot{e}_i(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ C_2 & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ e_i(t) \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} s(t) + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w(t), \\
z(t) &= \begin{bmatrix} C_1 & 0 \\ 0 & \beta I \end{bmatrix} \begin{bmatrix} e(t) \\ e_i(t) \end{bmatrix} + \begin{bmatrix} D_{12} \\ 0 \end{bmatrix} s(t) + \begin{bmatrix} D_{11} \\ 0 \end{bmatrix} w(t), \\
y(t) &= \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} e(t) \\ e_i(t) \end{bmatrix} + \begin{bmatrix} D_{21} \\ 0 \end{bmatrix} w(t).
\end{aligned} \tag{17}$$

Defining $e_a = [e \; e_i]^T$ then we can rewrite the first equation of (17) as

$$\dot{e}_a(t) = A_a e(t) + B_{2a} s(t) + B_{1a} w(t), \tag{18}$$

We notice that to use the integral action in the controller it was necessary to augment the system. However, considering only the augmented system, the integral action doesn't affect the controlled output $z(t)$, since the integral error state isn't observable by the considered performance output, and the calculated gains referring to the integral action are very small and little affect the system. Therefore, a small β term is used such that the integral term is observable in the controlled output $z(t)$, and generates integral gains that make a difference in the controller, while slightly modifying the traditional result of $\mathcal{H}_\infty$. As can be seen, (15) and (18) share the same structure, and thus, we can use Theorem (3) to design a controller that asymptotically stabilizes (15) with a $\mathcal{H}_\infty$ guaranteed cost γ and an included integrator to guarantee that the steady-state tracking errors vanish.

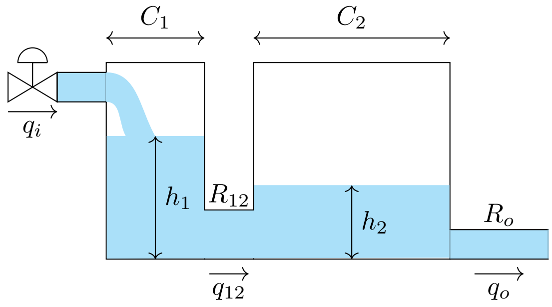

Figure 2. Interacting tank system.

5. APPLICATION EXAMPLE

To illustrate the proposed approach, consider the following simulation results. We computed the controller gains with the aid of Matlab, using YALMIP (Löfberg, 2004) as parser and the semi-definite programming tool Mosek (Andersen and Andersen, 2000) as solver. Figure 2 illustrates the level control of the interacting tank system whose linearized state representation is given by

$$\begin{cases}
\dot{x}(t) = \begin{bmatrix} -0.50 & 0.500 \\ 0.250 & -0.275 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t), \\
z(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t), \\
y(t) = [0 \;\; 1] \, x(t).
\end{cases} \tag{19}$$

The goal is to design a dynamic output feedback controller to track the continuous-time reference model :

$$G(s) = \frac{a_1 a_2}{(s + a_1)(s + a_2)}, a_1 = -3.8, a_2 = -0.08, \tag{20}$$

and to reject the disturbance. The disturbance was considered as the input flow rate of tank 1. The following results were obtained when we use an approach with and without integral action (using $\beta = 0.001$):

- Reference and state feedback gain: $\hat{K}_r = 6.25$ and $\hat{K} = [15.50 \; -11.05]$.
- Without integral action - optimal value: $\gamma = 0.01$
$$J_1 = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = 10^{10} \times \begin{bmatrix} -0.13 & -0.46 & 0.00 \\ -0.00 & -0.00 & -0.00 \\ 0.83 & 2.86 & -0.01 \end{bmatrix}.$$
- Without integral action - sub-optimal value : $\gamma = 0.4$
$$J_2 = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} -36.32 & -138.77 & 12.19 \\ 5.08 & -11.00 & -68.59 \\ 200.22 & 742.80 & -140.25 \end{bmatrix}.$$
- With integral action - optimal value: $\gamma = 0.008$

$$J_3 = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}$$
$$= 10^7 \times \begin{bmatrix} -0.08 & -0.32 & 0.00 & -0.01 & -0.00 \\ 0.00 & -0.00 & -0.00 & -0.16 & -0.00 \\ -0.00 & 0.00 & -0.00 & 0.00 & -0.00 \\ 0.53 & 2.00 & -0.00 & -0.02 & 0.00 \end{bmatrix}.$$

- With integral action - sub-optimal value: $\gamma = 0.4$

$$J_4 = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}$$
$$= \begin{bmatrix} -32.42 & -117.61 & 0.31 & 9.04 & -0.15 \\ 5.08 & -8.22 & -0.30 & -63.54 & -1.37 \\ -0.02 & 0.35 & -1.09 & -1.70 & -2.91 \\ 177.51 & 623.86 & -1.63 & -121.29 & -0.10 \end{bmatrix}.$$

Figure 3 displays the simulation results for the interacting tank system when we use an approach with and without integral action. As we can see in Figure 3, the optimal value to γ found when we used the with action integral approach was $\gamma = 0.008$. However, in this case, the gains were extremely high, on the order of 10^{10}, as also can be seen for the controller without integral control. As we know, extremely high gains are not suitable in practical situations. Therefore, the aim of Figure 3 is to show that even using a sub-optimal controller, it was possible to obtain perfect output tracking and disturbance rejection. The same was not obtained without integral action, since disturbance rejection only occurs when extremely high gains were used, which is impractical, as previously reported. As shown in Figure 3, the controller with integral action perfectly tracks the reference model transient in the presence of a load disturbance.

Figure 4 is concerned with the system behavior when a sinusoidal disturbance of amplitude 1, frequency 0.5 rad/s is applied. In this case, for both approaches, perfect output tracking and disturbance rejection only happen when we used high gains. The main advantage of this paper is a strategy based on two degrees of freedom structure with control action that guarantees output tracking and disturbance rejection. Moreover, the controller synthesis is based on a two-step LMI procedure. Figure 5 shows the error between plant and model outputs. A performance quantification of the controlled system behavior can be made by ISE (Integral of the square of the error), which is defined as (Dorf and Bishop, 2008):

$$ISE = \int_0^t e_y^2(\tau) d\tau. \quad (21)$$

This index is used because it is directly related to what is calculated in this work since the objective is to minimize the $\mathcal{H}_\infty$ norm. The results are summarize in Table 1.

Table 1. ISE performance index

Structure	Disturbance	γ	ISE
With integral control	Load	Optimal	0.004
Without integral control		Optimal	0.007
With integral control		Sub-optimal	0.657
Without integral control		Sub-optimal	10.580
With integral control	Sinusoidal	Optimal	0.002
Without integral control		Optimal	0.003
With integral control		Sub-optimal	3.588
Without integral control		Sub-optimal	3.525

Looking at Table 1, we notice the error of the structure with integral action for the sub-optimal case and a load disturbance is much smaller compared to the structure without integral action. It is important to remark that, according to the internal model principle, the augmentation of the system with integrators for tracking requires introducing in the system a model of the class of signals that the system will track (Eugene and Kevin A, 2013). Regarding sinusoidal disturbance, Table 1 reveals no clear advantage with and without integral action, both structures attenuated the disturbance.

6. CONCLUSION

This article proposed an approach to 2DOF controller design, in a decoupled structure, for linear continuous-time systems. The structure proposed uses one degree to asymptotic track the output of specified model reference, which contains all performance desired to the system. This performance is obtained using pole placement by state feedback. The other degree of freedom is used for bounded disturbance rejection by dynamic output feedback with $\mathcal{H}_\infty$ performance and integral action. Based on an augmented state space, that includes the output error dynamic between system and model reference, the synthesis procedure of $\mathcal{H}_\infty$ gain is presented in the form of linear matrix inequalities (LMI), that is solved in a two-step procedure, solvable by methods in the literature. The example shows that the proposed controllers guarantee a proper disturbance rejection and convergence of the error between the outputs of the reference model and plant. This proposed approach seems promising and as future research, we are interested in extending these results for nonlinear plants described by LPV or Takagi-Sugeno models and include an adaptive control strategy, to cope with uncertainty and possible time-varying parameters in the plant.

7. ACKNOWLEDGMENTS

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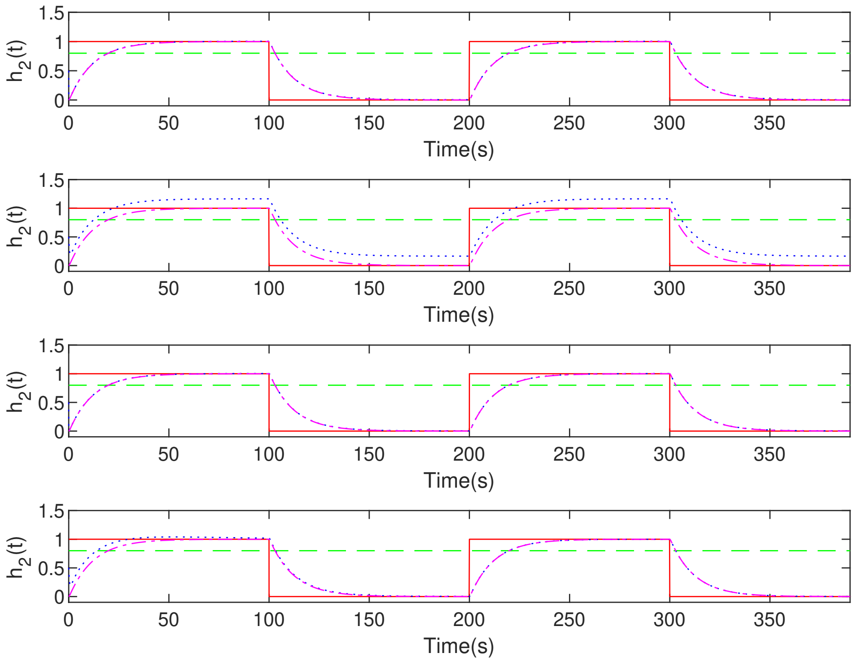

Figure 3. Transient responses of the reference (solid line), plant output, $h_2(t)$ (dotted line), of the reference model (dash-dotted line) and external disturbance w (dashed line) for controller without (J_1 and J_2) with integral (J_3 and J_4) control in sequence, when a load disturbance of amplitude 0.8 is presented.

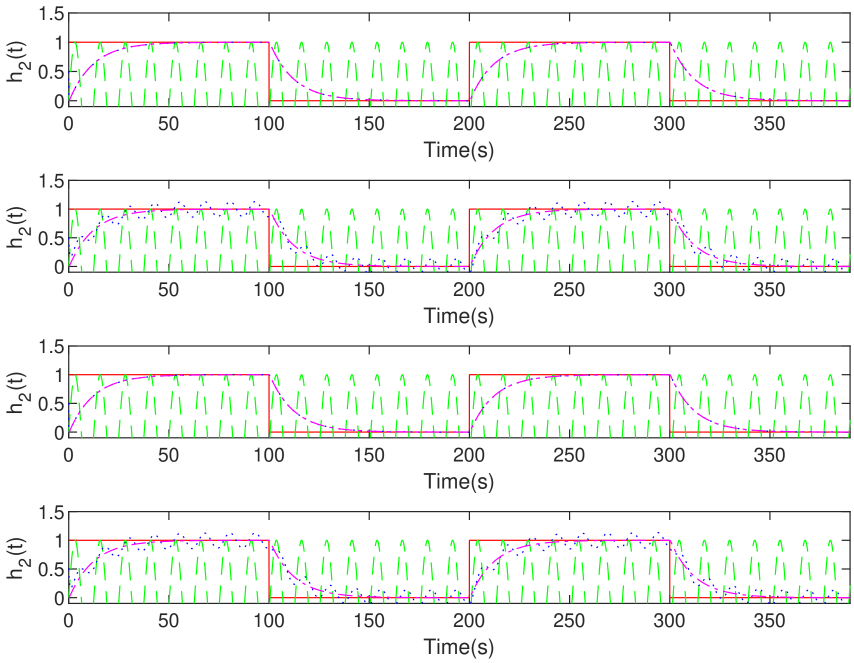

Figure 4. Transient responses of the reference (solid line), plant output, $h_2(t)$ (dotted line), of the reference model (dash-dotted line) and external disturbance w (dashed line) for controller without (J_1 and J_2) with integral(J_3 and J_4) control in sequence, when sinusoidal disturbance of amplitude 1 and frequency 0.5 rad/s.

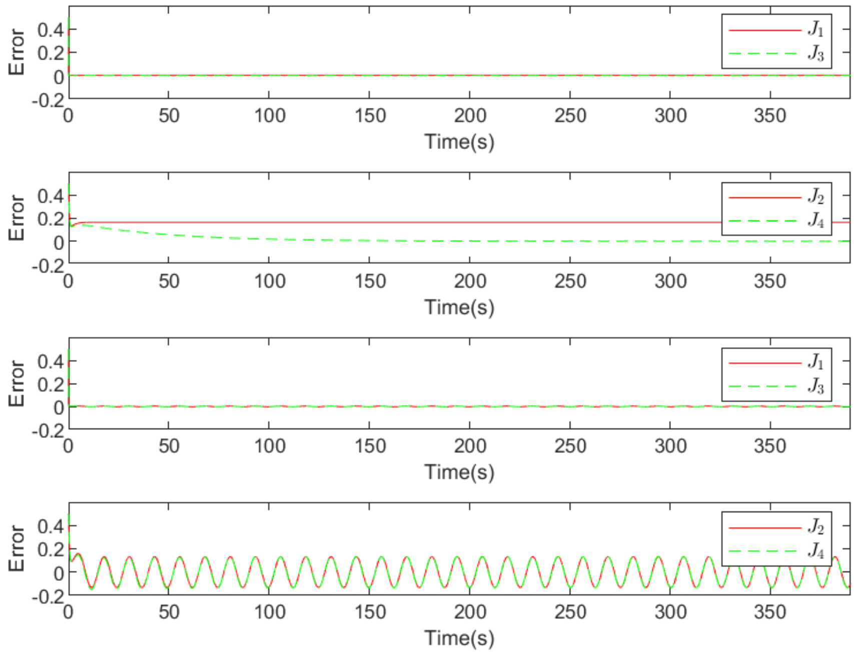

Figure 5. Transient responses of the error between plant and model outputs for the structure with (dashed) and without (solid line) integral control when load disturbance of amplitude 0.8 (top two panels) and sinusoidal disturbances of amplitude 1, frequency 0.5 rad/s, (bottom two panels) are applied.

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