Characterization of Distribution Systems Topological Flexibility using Bipartite Multigraphs

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Abstract: Over recent decades, power distribution systems have been required to handle extreme events that are increasing in both frequency and intensity. In addition, there are novel technologies, planning strategies, and operational approaches. In this context, topological flexibility is a key aspect to allow distribution systems to accommodate all the simultaneous emerging changes and requirements. Topological flexibility is the capability of a system to rearrange its structure and is directly related to sectionalizing and tie switches operation. In this paper, metrics calculated from a bipartite multigraph representation aiming to describe the topological flexibility of distribution systems are presented. Different distribution systems are used to illustrate the bipartite graph representation and the derived metrics. Our approach is directly applicable to distribution systems characterization and enables future development of metrics to fully describe the topological flexibility of such systems.

Resumo: Nas últimas décadas, os sistemas de distribuição de energia têm sido submetidos a eventos extremos que estão aumentando em frequência e intensidade, além disso, novas tecnologias, estratégias de planejamento e abordagens operacionais vem sendo colocadas em prática. Nesse contexto, a flexibilidade topológica é um aspecto fundamental para permitir que os sistemas de distribuição acomodem todas as mudanças e requisitos que emergem simultaneamente. A flexibilidade topológica é a capacidade de um sistema rearranjar sua estrutura e está diretamente relacionada a operação das chaves. Neste trabalho, são apresentadas métricas calculadas a partir de uma representação por multigrafos bipartidos com o objetivo de descrever a flexibilidade topológica de sistemas de distribuição. Diferentes sistemas de distribuição são utilizados para ilustrar a representação por multigrafos bipartidos assim como as métricas apresentadas. A abordagem relatada neste trabalho é diretamente aplicável à caracterização de sistemas de distribuição e possibilita o desenvolvimento futuro de métricas para descrever completamente a flexibilidade topológica de tais sistemas.

Keywords: Modeling and Simulation of Power Systems; Power Distribution Systems; Complex Adaptive Systems; Topological Flexibility; Bipartite Graphs.

Palavras-chaves: Modelagem e Simulação de Sistemas Elétricos de Potência; Sistemas de Distribuição de Energia; Sistemas Adaptativos Complexos; Flexibilidade Topológica; Gráfos Bipartidos.

1. INTRODUCTION

Power distribution systems, like other engineering systems, are exposed to random extreme events that are increasing in both frequency and intensity. One manner to manage the outages caused by such events is by operating manual or remote-controlled sectionalizing and tie switches to, after identification and isolation of faults, restore the healthy out-of-service consumers followed by maintenance of damaged parts (Arif et al., 2017). This service restoration process is of central importance to achieving higher reliability (Xu et al., 2015) and resiliency (Haggi et al., 2019) of distribution systems.

Another relevant aspect is the uncertainty associated with the penetration of intermittent distributed and renewable energy sources (RES) in electric power distribution systems, aiming to reduce costs, losses, and emissions (Alam and Arefifar, 2019). Despite the proposals of combining RES with energy storage solutions such as batteries or electric vehicles (Malya et al., 2021), the need to accommodate the uncertainties is fundamental, which can be performed by adaptive system reconfiguration through the operation of sectionalizing and tie switches (Liu and Srikantha, 2021). Moreover, these RES features also need

to be considered during service restoration (Faria et al., 2021).

As indicated in the preceding paragraphs and foretold by Brown (2008), this topological flexibility, i.e., the capability to reconfigure the system structure by switch operations as a response to different needs, plays a central role in the operation, management, and design of power distribution systems, enabling strategies to deal with traditional and novel challenges.

However, the majority of studies on the flexibility of power systems (Zhao et al., 2016; Degefa et al., 2021) are focused on managing the uncertainties in both renewable generation and increasing demand. Recently, Gu and Chu (2020) presented an approach to quantify the topological flexibility of power distribution systems based on graph communities detection. It considers connectivity between elements in the distribution systems to group them as communities and quantifies the intra-groups and inter-groups connections; despite tie switches accounting, electrical features are not considered.

In this paper, the use of a bipartite multigraph representation of power distribution systems to quantify these systems' topological flexibility is presented. The bipartite multigraph embeds the operationally feasible paths between all pairs of load and source buses, and these paths are a consequence of the different manners to combine sectionalizing and tie switches status. Given the bipartite multigraph, metrics describing the arrangement of the feasible paths can be calculated to quantify the system's topological flexibility in a direct manner.

Bipartite representation has been used in power systems problems before, for reliability evaluation (Faza et al., 2007), fault detection and location (Dustegor et al., 2009), service restoration (Košt'álová and Carvalho, 2011), and recently for management of power exchange between electrical vehicles (Zeng et al., 2020). Since it is a graph-based representation, it allows the direct use of well-known and easily interpretable metrics. Moreover, this bipartite representation can be constructed by combining structural and electrical features that affect the value of the metrics.

The remainder of this paper is organized as follows: Section 2 presents the fundamentals of Bipartite graphs and how they can be used to represent a power distribution system. In Section 3, the metrics that are useful to quantify topological flexibility are defined. Section 4 outlines the experiments and the different distribution systems used to illustrate our approach. In Section 5, the results and discussion are presented, and Section 6 concludes this manuscript.

2. BIPARTITE REPRESENTATION

A bipartite graph is a triple $G = (U, V, E)$, where U and V are two disjoint sets of vertices, and $E = \{(u, v) : u \in U, v \in V\}$ is a set of edges connecting vertices between U and V. The main difference between a bipartite graph and a classical graph representations (Guillaume and Latapy, 2004) is that vertices in U only can be connected to vertices in V, and vice-versa. A natural example of such an organization is individuals-events networks (Scott and Carrington, 2011). In this work, a bipartite multigraph (Abbas and

Hong, 2013), which can have multiple edges connecting the same vertices pair, is used to represent power distribution systems.

The representation of a distribution system as bipartite multigraph adopted here considers the energy source buses as vertices in U and load buses as vertices in V. An edge (u, v) connecting a vertex $u \in U$ and a vertex $v \in V$ will exist for each path between the two buses in the distribution network that respect the operating constraints of the system when only this connection is considered. In this manner, the bipartite graph will represent the different manners of connecting load buses to source buses by considering the paths formed by the combination of all switches present in the distribution system.

A usual representation of the system introduced by Civanlar et al. (1988) is presented in Fig. 1, with the open and closed switches that can operate to rearrange the system topology. The bipartite multigraph representation of this system is illustrated in Fig. 2. All the load buses have three different manners to be connected to some source bus; in other words, there exist combinations of switches status, which allow feasible connections of each source bus to all the load buses. These connections are evident from the bipartite representation, and highlight the topological flexibility of the system.
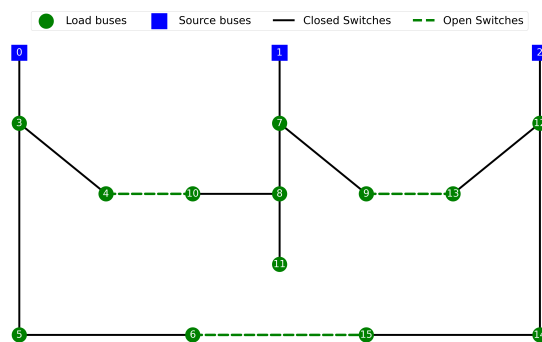

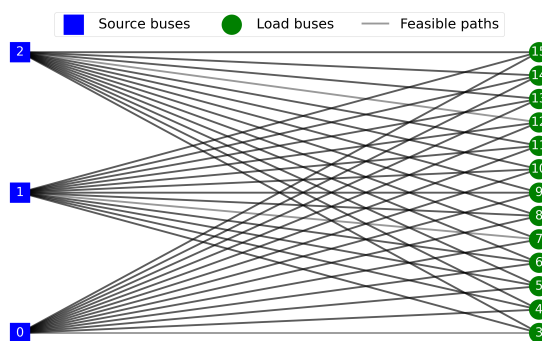Figure 1. Usual representation of a power distribution system.

Figure 2. Bipartite representation of the power distribution system in Figure 1.

3. QUANTIFYING TOPOLOGICAL FLEXIBILITY

Although the differences between a graph and a bipartite graph as illustrated at the beginning of the previous

section, graph metrics that consider only topological aspects (Costa et al., 2007) or hybrid ones, which embed electrical features of power systems during metric calculations (Cuadra et al., 2015), can be used to characterize bipartite graphs. A metric that can be used to characterize the vertices of a graph is the *degree*, which quantifies the number of connections a vertex has. For a bipartite graph (Scott and Carrington, 2011), the normalized degree d of a vertex is calculated as:

$$d_v = \frac{\deg(v)}{m}, \quad \text{for } v \in V, \tag{1}$$

$$d_u = \frac{\deg(u)}{n}, \quad \text{for } u \in U, \tag{2}$$

where m and n are the number of vertices in sets V and U, respectively. By applying this metric, it is possible to quantify how connected the vertices of the two sets are and also evaluate the overall connectivity on the bipartite multigraph.

For the bipartite representation of a distribution system described in Section 2, d_v will describe the number of source buses that can be used to serve the load bus v, and d_u will quantify the proportion of load buses that can be served by the source bus u. If a load bus v has two or more edges to the same source bus in the bipartite multigraph representation, d_v can be higher than one. A direct consequence of using d_v and d_u to characterize the distribution system from the bipartite multigraph representation is that a higher d_v reflects a higher number of alternative paths to connect a load bus to source buses, i.e., the distribution system has topological flexibility to be reconfigured. A higher d_u indicates that a source bus can be used to serve various load buses. In both cases, a higher d will reflect higher topological flexibility.

Despite that, d_u and d_v do not account for specific features of a power distribution system. This limitation can be overcome by incorporating some electrical features as weights associated to edges. Here, the sum of the power served to each load bus in the feasible path represented by an edge (u, v) in the bipartite multigraph is used as the edges' weight. In this manner, the weighted degree deg^w of each node in the bipartite multigraph representation will be the sum of the edges' weight connected to it, and the normalization factor will be the total power (TP) that must be served in the distribution system:

$$d_v^w = \frac{\deg^w(v)}{TP}, \quad \text{for } v \in V, \tag{3}$$

$$d_u^w = \frac{\deg^w(u)}{TP}, \quad \text{for } u \in U, \tag{4}$$

in which d_u^w and d_v^w are the normalized weighted degrees of source and load buses, respectively.

In this manner, a higher d_u^w will indicate that a source bus can serve a high power demand through different paths. On the other hand, a higher d_v^w will suggest that a load bus can be served by different paths that attend to a significant part of the overall distribution system demand.

4. TEST CASES

A set of distribution systems will be used to illustrate this topological flexibility quantification based on the bipartite multigraph representation. They are presented in Table 1 with their respective references.

Table 1. Distribution systems used in this study. The triples (S, L, T) indicate the number of source buses, load buses, and tie switches, respectively.

System	(S, L, T)	Reference
#1	(3, 13, 3)	(Civanlar et al., 1988)
#2	(1, 29, 1)	(Eminoglu and Hocaoglu, 2005)
#3	(1, 32, 5)	(Baran and Wu, 1989)
#4	(11, 83, 13)	(Su et al., 2005)

System #1, illustrated in Fig. 1, #3, and #4 were introduced in distribution system reconfiguration studies for loss reduction, a situation where topological flexibility is fundamental. System #2, introduced in a power flow study, is the only one presented in a context where topological flexibility was not relevant, having the lower number of tie switches.

The bipartite multigraph representation of the four systems presented in Tab. 1 is obtained by evaluating the feasibility of all the independent paths between each pair u, v. An edge representing a path is added in the bipartite multigraph representation if the voltage magnitude of all buses in this path respects the following condition:

$$V^{\min} \le V_i \le V^{\max}, \tag{5}$$

where V_i are the i-th bus in the path connecting a source u to a load bus v, and $V^{\min}$ is 0.95 p.u. and $V^{\max}$ is 1.05 p.u. The power flow calculations were performed using the Python package *pandaspower* (Thurner et al., 2018).

The normalized degrees, d_u and d_v, and the weight normalized degrees, d_u^w and d_v^w, will be calculated for the obtained bipartite multigraphs. These values will be evaluated using graphics and summarized by basic statistics of the observed values of unweighted and weighted degrees.

In addition, an experiment related to service restoration using the systems listed in Table 1 will be conducted. This experiment will consist of solving 30 service restoration situations where the number of faulted buses is the integer closest to 10% of the amount of load buses of each system. After obtaining a solution to the service restoration problem, the final % of the Complex Power (S) Not Served (SNS) without considering the faulted buses will be computed. This SNS will be compared with the unweighted and weighted degrees of each system to evaluate how these bipartite graph metrics are related to the topological flexibility of each system. The service restoration problem will be solved using the meta-heuristic approach presented by Goulart et al. (2018) and considering the voltage requirement for each load bus equal to the one presented in (5).

5. RESULTS AND DISCUSSION

The first result to be presented is for the System #1, illustrated in Figs. 1 and 2. The normalized unweighted degrees, d_u and d_v, and weighted degrees, d_u^w and d_v^w, values are presented on histograms in Fig. 3. The d_u values are 1.92 for the three source buses, resulting from 25 different paths between each source bus and the 13 load buses. The d_v values are 2 for 10 load buses, and 1.67 for

the others 3 load buses. These values indicate that all the load buses have more than 3 paths to the source buses. The results are different when compared to the weighted degrees: d_u^w is different for the three source buses, and d_v^w has more variability than its unweighted version. These values indicate that the source buses can serve a large amount of the loads' power demand through a high amount of alternative paths, reflecting the topological flexibility of System #1.
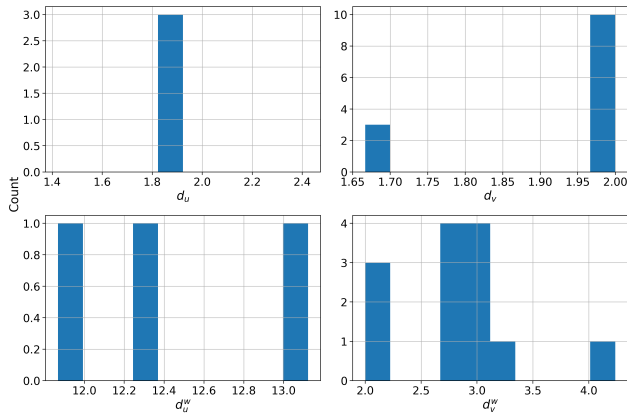

Figure 3. Histograms of d_u, d_v, d_u^w, and d_v^w values for System #1.

Figure 4 presents the values for System #2. d_u are 1.19 for the only source bus this system has, i.e., it has 31 feasible paths for all the load buses. d_v values are 0 for 4 load buses, indicating that the paths between these load buses and the source bus are not feasible due to the condition in Eq. 5, 13 load buses have 1 feasible path, and 9 load buses have 2 feasible load paths to the source bus. Differently from System #1, System #2 has a d_u value near one, and only nearly one-third of the load buses have alternative paths to the source bus. In addition, besides some values of d_v^w near unity, a majority of buses resulted in d_v^w lower than 0.35, indicating that the feasible paths can serve only 35% of the load buses demand. This fact reflects reduced topological flexibility, which was not necessary for the context where such a system was introduced.
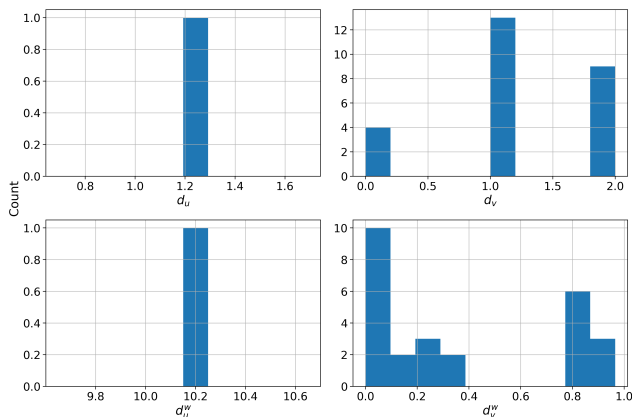

Figure 4. Histograms of d_u, d_v, d_u^w, and d_v^w values for System #2.

The values for System #3 are presented in Figure 5. d_u is 8.68, indicating that the only source bus has 278

paths for all the 32 load buses. Differently from the two previous systems, this one presents a higher variability of d_v, it ranges from 1 to 15, and the d_v value with higher occurrence is 9 for 6 load buses. An important aspect here is that this system has an average d_v much higher than the observed for Systems #1 and #2. d_v^w also presents high values, with most being higher than one. These high values of unweighted and weighted degrees reflect the higher topological flexibility of this system when compared with System #2.
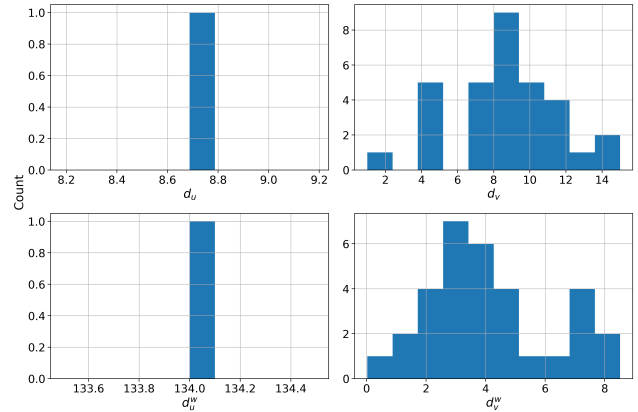

Figure 5. Histograms of d_u, d_v, d_u^w, and d_v^w values for System #3.

The values for System #4, which is the system with the highest number of source and load buses and tie switches, are presented in Figure 6. d_u ranges from 0.04 for source bus 9 to 0.98 for source bus 3, a 24-fold difference. This point reflects the difference in the capability of source buses to accommodate load buses during some topological rearrangement of this system. The average d_u value is 0.46, indicating the existence of 339 different feasible paths between all pairs of source and load buses.
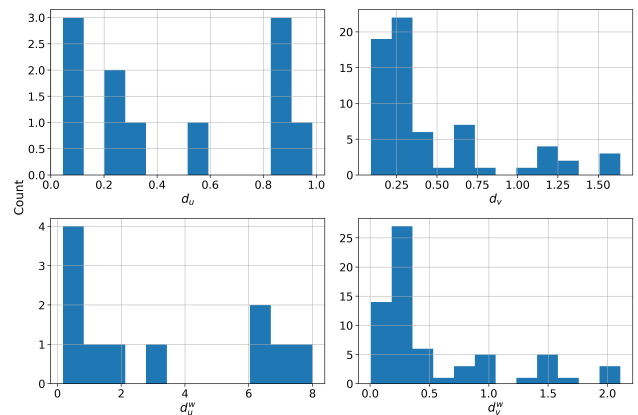

Figure 6. Histograms of d_u, d_v, d_u^w, and d_v^w values for System #4.

Similar to System #3, d_v also has significant variability for System #4, but with lower values. The value with higher occurrence is 0.27 for 22 load buses, and the average d_v is 0.46, which is the lowest average d_v observed for the four systems. Despite these low unweighted degree values, the d_v^w are not the lowest ones observed, being higher than those observed for System #2. The d_u^w values are also

similar to the ones observed for System #1, which is the other System with more than one source bus, but here the capacity of source buses to meet a higher amount of load buses demand has an expressive variability.

Table 2 summarizes the results by presenting the average values for the d_u, which is equal to the average of d_v, and the average values of d_u^w and d_v^w, observed for the four systems presented in Table 1. These average values show that System #4, which has the lowest unweighted degree, will not necessarily have the lowest weighted degree for load buses. Another aspect is that systems with equal unweighted degrees can have different weighted degrees, such as Systems #1 and #2. This highlights the importance of evaluating the structural and electrical features of distribution systems when they are being characterized using the bipartite multigraph representation presented here.

Table 2. Summary of unweighted, d_u and d_v, and weighted, d_u^w and d_v^w, degrees observed for the four systems presented in Table 1.

System	$\bar{d}_u = \bar{d}_v$	$\bar{d}_u^w$	$\bar{d}_v^w$
#1	1.92	12.42	2.86
#2	1.92	10.15	0.39
#3	8.68	133.99	4.18
#4	0.46	3.23	0.54

The results of the service restoration experiment are presented in Figure 7, and the median SNS (%) observed is shown in Table 3. The first aspect that must be mentioned is that Systems #1 and #4 always restored some of the healthy out-of-service loads; while Systems #2 and #3 resulted in trials where 100% of the healthy out-of-service loads were not restored, indicated by the SNS equal to 1 in the histograms of Figure 7.
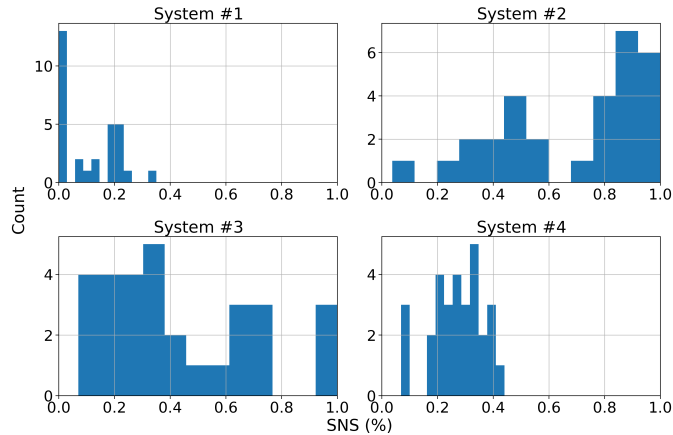

Figure 7. Histogram of SNS (%) for the 30 service restoration experiments with the four systems.

These results suggest System #1 as having the highest topological flexibility, followed by System #4; while System #2 is the one with the lowest topological flexibility. By considering the values in Table 2, higher unweighted and weighted degrees values seem to quantify a fraction of the topological flexibility, but there is one obvious factor that is not accommodated in these degrees, namely the number of source buses that can be used during a situation that requires topological flexibility such as service restoration.

Table 3. Median values of the observed SNS Summary of the service restoration experiments using the four systems presented in Table 1.

System	Median SNS
#1	8.46%
#2	75.30%
#3	27.39%
#4	28.38%

Examining the results for the Systems #2 and #3, which are the ones with only one source bus, the #3 is the one with the highest $\bar{d}_v^w$ and $\bar{d}^v$. The same can be observed for the Systems #1 and #4, which are the two with more than one source bus. System #1 with higher $\bar{d}_v^w$ and $\bar{d}^v$ exhibited a better result in the service restoration experiment. Another important aspect is that by using these degrees, it is possible to evaluate which load buses have fewer feasible paths to the source buses and which source buses could be made more available to serve the load buses. This information can be used during the decision for allocation of switches to increase the topological flexibility.

6. CONCLUSION

Power distribution systems are constantly exposed to different threats that can result in service interruption, and service restoration capability is fundamental to achieving a reliable and resilient operation. In addition, renewable sources are being integrated with these systems, which also demands a capability to reconfigure the system through switches operation to accommodate the inherent variability of renewables. In both cases, topological flexibility is a fundamental aspect to achieve better electricity delivery to customers.

The power system flexibility literature is engaged in managing the uncertainty in the increasing demand and renewable sources penetration. Besides this important aspect, topological flexibility, which is related to the capability of the system to reconfigure its structure to achieve a better operational status in different situations, has few studies. In this context, this manuscript presented the use of a bipartite multigraph representation to quantify some properties that are related to the topological flexibility of power distribution systems.

The bipartite multigraph represents the paths between source and load buses that are feasible as edges, allowing direct quantification of how many different paths exist between each pair of source and load buses. Moreover, by accounting for the total power served at each path is possible to describe the amount of power demand that can be met by using each one of the feasible paths. These characteristics can be quantified by using unweighted and weighted degrees for the source and load buses.

The metrics and the service restoration experiments with the four distribution systems indicated that there is a relationship between the observed degrees and the service restoration quality, which is directly related to the topological flexibility of distribution systems. A limitation of this study is the use of only small distribution systems, due to the computational costs of evaluating the feasibility of all

the existing paths between each pair of source and load buses in larger systems.

Despite this, the results indicate that the use of bipartite multigraph representation can bring important information related to the topological flexibility of power distribution systems. Future research will expand the experiments with larger distribution systems, include systems with distributed generation, and evaluate other metrics extracted from the proposed bipartite representation. Such an approach can result in useful metrics to assist in the various emerging challenges on distribution systems, as reliability and resilience issues.

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