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Optimal Economic Dispatch of Thermoelectric Power Units with Practical Constraints Through an Enhanced Bat Algorithm

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Abstract: The thermal Economic Dispatch (ED) consist of a complex optimization problem of power systems planning and operation that aims to minimize the overall generation cost. The realistic representation of the thermoelectric power units is modeled by practical constraints comprising valve-point effect, multiple fuels and Prohibited Operating Zones (POZ), which makes the ED a nonconvex and discontinuous mathematical programming problem. This paper is proposes an Enhanced Bat Algorithm (EBA) to solve the ED problem with its practical constraints. The EBA is developed to improve the exploration of the search space through some modifications in the original Bat algorithm (BA), with the purpose of increasing the efficiency in finding good solutions for large scale ED problems. Case studies with statistical analysis are performed to assess the proposed approach, involving a system with all practical constraints related to the ED problem, which is a contribution of the present paper.

Keywords: Economic Dispatch; Practical Constraints; Enhanced Bat Algorithm; Nonconvex; Discontinuous.

1. INTRODUCTION

Nowadays, in face of the increase in the fossil fuels costs and the environmental impact issues, as well as the increase in the load demand of Electrical Power Systems (EPS), the optimization of the thermoelectric generation becomes even more necessary improve the demand supply with minimal cost. This optimization problem is know as Economic Dispatch (ED) and aims at minimizing the total cost of power generation, which comprises the fuel and operation costs.

The input and output characteristics of thermoelectric units has been investigated in the literature, as in Wood and Wollenberg (1996) where an approximated quadratic function that associates the active power dispatch with fuel and operation costs is used. However, in the practical ED problem, other factors directly affect the input-output relation, such as the valve-point effect, the Prohibited Operating Zones (POZ) in addition to multiple fuel constraints that make the ED a nonconvex, discontinuous and complex mathematical programming problem.

To solve the complex aforementioned ED problem, several optimization techniques based on metaheuristics have been used, mainly the bio-inspired computing, whose algorithms are investigated for application, either in their original versions or with specific improvements (Kar, 2016). Among the methods in the original forms, Genetic Algorithm (GA)(Chiang, 2005), Differential Evolution (DE)(Sayah and Hamouda, 2012), Artificial Bee Colony (ABC)(Hemamalini and Simon, 2010), Biogeography - Based Optimization (BBO)(Bhattacharya and Chattopadhyay, 2010b), Particle Swarm Optimization (PSO)(Selvakumar and Thanushkodi, 2008) and Krill Herd Algorithm (KHA) (Mandal et al., 2014) have been applied to solve ED problem. Some improvements in the previous methods are the Anti - predatory PSO (APSO)(Selvakumar and Thanushkodi, 2008), Adaptive PSO (APSO)(Panigrahi et al., 2008), the Modified PSO (MPSO) (Basu, 2015) and the Enhanced Differential Evolution (EDE)(Sayah and Hamouda, 2012).

Moreover, hybrid optimization techniques that combine different metaheuristic methods have been proposed to take advantage of the good search mechanisms of each one in a more efficient manner. Among the hybrid approaches that have been applied to the ED problem, it can be cited the Improved Genetic Algorithm with Multiplier Updating (IGA_MU)(Chiang, 2005), New Particle Swarm Optimization with Local Random Search (NPSO_LRS)(Selvakumar and Thanushkodi, 2007), Orthogonal Design with Particle Swarm Optimization (IODPSO) (Qin et al., 2017), Differential Evolution with Biogeography-based Optimization (DE/BBO)(Bhattacharya and Chattopadhyay, 2010a) and the Real-Valued Mutation (RVM) operating into the PSO (PSO-RVM) (Lu et al., 2010). The state-of-art includes other nature-inspired techniques that have also been applied to solve ED problem, as the Backtracking Search Algorithm (BSA) (Modiri-Delshad et al., 2016) and the Kinetic Gas Molecule Optimization (KGMO) (Basu, 2016).

In this line of research the present paper proposes the application of a new Enhanced Bat Algorithm (EBA) to solve the optimization problem of ED, taking into account the practical constraints related to this problem in a coupled manner. The proposed EBA improves the search process with a local search strategy what makes the optimization procedure more efficient. Additionally, the result obtained by EBA is compared with others from technical literature and with the original BA and the the efficiency of the proposed approach is assessed by a statistical analysis.

The subsequent sections are subdivided in as follows: section 2 presents a brief description of the ED problem with its mathematical formulation. In section 3 are described the proposed Enhanced Bat Algorithm. The results and discussions of case studies with the statistic analysis of the proposed approach are presented in section 4, and the main conclusions of the paper is presented in section 5.

2. THE ECONOMIC DISPATCH PROBLEM

In the present paper the proposed mathematical formulation for this problem has an objective function that comprises the fuel cost functions of all power units of the system, and constraints that includes: (i) the active power balance equation involving the total power generation, the load demand, and the transmission loss; (ii) the maximum and minimum power generation limits (Wood and Wollenberg, 1996); and (iii) prohibited operating zones. Moreover, the proposed model considers in its objective function the valve-point effect and multiple fuels. Therefore the ED problem can be formulated as follows:

$$\begin{aligned} \text{Minimize} \qquad & F_T = \sum_{i=1}^{n_g} F_i(P_i) \\ \text{subject to:} \qquad & \sum_{i=1}^{n_g} P_i = P_D \\ & P_i^{\min} \leq P_i \leq P_i^{\max} \end{aligned}$$

where:

- F_T : Total generation cost;
- F_i : Cost function of thermoelectric unit i;
- P_i : Power of thermoelectric unit i;
- P_D : Total system load demand;
- P_L : Total transmission loss;
- $P_i^{\min}$: Minimum output of thermoelectric unit i;
- $P_i^{\max}$: Maximum output of thermoelectric unit i;
- n_g : Number of thermoelectric units of the power system.

2.1 Approximated cost function

The fuel cost function relates the nonlinear dependence between its input (fuel) and output (electrical power), as formulated by the quadratic function of equation (1) (Wood and Wollenberg, 1996):

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{1}$$

where:

- a_i, b_i, and c_i: Cost coefficients of thermoelectric unit i.

2.2 Constraints

The constraints of the ED problem refer to the limitations of the thermoelectric generations units and the operation of the power system that must supply its load demand reliability and continuity. The constraints considered are described hereafter:

Power Balance: In this paper, refers to the balance between the load demand (P_D) and supply through the total generation of electrical power, as follows:

$$\sum_{i=1}^{n_g} P_i = P_D \tag{2}$$

Generation Limits: The generation limits are associated with mechanical limitation of the equipment (Zhu, 2015) and are represented by equation (3).

$$P_i^{\min} \leq P_i \leq P_i^{\max} \tag{3}$$

Valve-point: This effect is related with temperature control and occurs as each steam admission valve in the turbine starts to open, implying in ripples in the cost curve, according to Chiang (2005). The inclusion of this constraint adds a higher other order nonlinearity thus a technique to analysis of local minimum has been proposed (Zhan et al., 2015a). This constraints is modeled by a sinusoidal functional, as described below:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 + \alpha_i \left| e_i \sin\left(f_i \left(P_i^{\min} - P_i \right) \right) \right| \tag{4}$$

where:

- e_i and f_i: Coefficients of generator i that represent the valve-point effect;
- α_i: Binary number that represents the selection of the valve-point effect.

Multiple Fuels: Represents the different type of fuel to operation that directly affect the operation costs of these thermoelectric units. Therefore the new cost function, considering the valve-point effect, for each thermoelectric unit (i) is modeled according to (Lin and Viviani, 1984; Bhattacharya and Chattopadhyay, 2010a; Park et al., 2010; Dieu and Schegner, 2013; Hemamalini and Simon, 2010), and presented in equation (5).

Prohibited Operating Zones (POZ) This constraint implicates in undefined regions of power output that producing discontinuities in the cost curves (Orero and Irving, 1996), as modeled by the following inequalities:

$$\begin{aligned} P_i^{\min} \leq\; & P_i \leq P_{i,1}^{l} \\ P_{i,j-1}^{u} \leq\; & P_i \leq P_{i,j}^{l} \qquad j = 2, ..., n_j \\ P_{i,n_j}^{u} \leq\; & P_i \leq P_i^{\max} \end{aligned} \tag{6}$$

where:

- $P_{i,j}^{l}$: Lower bound of prohibited operating zone.
- $P_{i,j}^{u}$: Upper bound of prohibited operating zone.
- n_j: Number of prohibited zones in thermoelectric unit i.

3. ENHANCED BAT ALGORITHM

The Bat Algorithm (BA) proposed by Yang (2010) is applicable in diverse areas of knowledge (Damodaram and Valarmathi, 2012; Kumaravel and Kumar, 2011; Kar, 2016), including the ED problem. In this paper is proposed modifications in the BA, which refer to local search and a percentage adjustment of the worst solutions, giving rise proposed method called Enhanced Bat Algorithm (EBA).

The operation characteristic of the ED problem is defined by bat that set the fuel and zone of operation of each thermoelectric unit, modeling the combinatorial problem due to inherent practical constraints. This information is sent to LINGO that return with the allocation of active power and the respective operation cost.

3.1 Procedures of the EBA

The premises of search process in this proposed method is similar to original BA, establishing a relation of the randomly moves. The adjust of position and velocity is based on frequency of the emitted pulses, which are represented by equations (7) to (9), according to Yang (2010).

$$f_i = f_{min} + (f_{max} - f_{min})\beta_i \tag{7}$$

$$v_i^{t+1} = v_i^t + (x_i^t - x_*)f_i \tag{8}$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \tag{9}$$

Where $\beta \in [0, 1]$ is a random scalar from an uniform distribution and x_* as the best bat of the population.

Aiming to improve exploration of the search space, both methods (BA and EBA) updates the parameters refer to a rate of pulse emission (r_i) and a loudness (A_i). In update occurs the decrease in the loudness, that is gradual, and increase in the rate of pulse emission that is related with iteration of the algorithm, as shown below:

$$r_i^{t+1} = r_i^0[1 - e^{(-\gamma t)}] \tag{10}$$

$$A_i^{t+1} = \alpha A_i^t \tag{11}$$

Where α and γ are positive constants within the intervals: $0 < \alpha < 1$ and $\gamma > 1$. In this way, when the optimal solution is approaching ($t \to \infty$), $A_i \to 0$ and $r_i \to r_i^0$.

The local search is based on the current best solution ($x_{old} = x_*$) of the population. In the EBA the analysis to realizes the local search is different of BA, according to Pseudocode 1. But the calculation of new solution is the same, as shown in (12), being ϵ a random number ($\epsilon \in [-1, 1]$) that varies for each bat in each iteration and loudness (A_i) of the respective bat in iteration t.

$$x_{i_{new}} = x_{old} + \epsilon A_i^t \tag{12}$$

3.2 Implementation of Enhanced Bat Algorithm

In the research of a method more efficient to solve ED problem, is proposed the EBA.Theses modifications and details of implementation are described in Pseudocode 1.

Pseudocode 1: Enhanced Bat Algorithm

Input: Parameters define: n, $f_{\min}$, $f_{\max}$, A, r^o, γ, α and PER
Create the bat population, x_i and v_i $(i = 1, 2, \ldots, n)$
Evaluate the bat through of the objective function ($F_T(x_i)$)
Find the best bat in population (x_*)
Setting the number of *worst* bats corresponding to rate PER

1 **while** (not meeting the convergence criterion) **do**
2 **for** (for each bat i) **do**
3 Update frequency, velocity, and position [(7) to (9)];
4 **if** $rand < r_i$ **then**
5 Do $x^t_{\text{old}} = x_*$;
6 Do local search [(12)];
7 **end**
8 **if** $x_i^{t+1} \in$ worst **then**
9 Get new values for this bat;
10 **end**
11 Evaluate the x_i^{t+1};
12 **if** rand $< A_i$ **or** $F_T(x_i^{t+1}) \leq F_T(x_*)$ **then**
13 Get the new solution;
14 Update r_i and A_i [(10) and (11)];
15 **end**
16 Get the best bat (x_*);
17 **end**
18 Update the iteration number ($t = t + 1$);
19 **end**

Output: The best bat (x_*)
Objective function value ($F_T(x_*)$)

The rate PER represents the number of worst bats of population. In analysis of each bat, when it is one of the worst its value is modifies randomly, but without leaving the feasible space, as show in line 8 to 10 in Pseudocode 1. Therefore, this rate PER inserts more flexibility in moves of the bats in search process, so the flexibility aggregate by rate PER improves the exploration the search space that is constitute by thermoelectric units of ED problem.

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 + \alpha_i |e_{i1}\ \sin(f_{i1}\,(P_i^{\min} - P_i))|, & \text{for fuel } 1, \quad P_i^{\min} \leq P_i \leq P_{i1} \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 + \alpha_i |e_{i2}\ \sin(f_{i2}\,(P_{i1} - P_i))|, & \text{for fuel } 2, \quad P_{i1} \leq P_i \leq P_{i2} \\ \quad \vdots \qquad\qquad \vdots & \qquad \vdots \qquad\qquad \vdots \\ a_{in_f} + b_{in_f}P_i + c_{in_f}P_i^2 + \alpha_i |e_{in_f}\ \sin(f_{in_f}\,(P_{i(n_f-1)} - P_i))|, & \text{for fuel } n_f, \quad P_{i(n_f-1)} \leq P_i \leq P_i^{\max} \end{cases} \tag{5}$$

where:
n_f : Total number fuel type of the thermoelectric Unit i.

The proposed modification in analysis of r_i, shown in line 4, increases the possibility of occurs the local search after some iterations, due to growth of r_i that is associated increase process iteration, as observed by (10). The other change adopted in EBA is related with the acceptance of the new solution and updates of the local search parameters, whose executions requires the attendance only one of the criteria, as described in line 12. The modifications results in efficient exploration of feasible space, which is evidenced on ED problem of larger systems, according to evaluations presented in the next section.

4. COMPUTATIONAL RESULTS AND DISCUSSIONS

This section presents two ED problem case studies that are solved by the original BA and the proposed EBA, your results are compared with the findings in the technical literature, being detailed and evaluated with the purpose of analyze the efficiency of each method.

Both methods are developed using MATLAB® on a personal computer with an Intel® Core™ i7 with 2.93 GHz and 8 GB RAM. To solve ED problem, the metaheuristics are used to sets the thermoelectric operation system refer to multiple fuels and operation zones, these information are sent to LINGO™ that realizes the optimization of the power generation of each thermoelectric unit. The parameters are set according to value of Table 1 and the convergence criterion is set in 100 iteration to both methods.

Table 1. Parameters of BA and EBA

$n = 25$	$r^o = 0.5$	$A = 0.25$
$f_{\min} = 0$	$f_{\max} = 2$	$\gamma = \alpha = 0.5$

4.1 System 1: 10 thermoelectric units with valve-point and multiple fuels

This system considering the number of 10 generation units with multiple fuels and valve-point effect (Chiang, 2005), the valve-point effect incorporation into the cost function implies in the addition of many local optima.The active power demand is set to 2700MW, and the power transmission loss has not been considered.

To suplly the demand the operation cost of Economic Dispatch is equal to 605.7 \$/h, for BA and EBA, which compared with recent literature it is among the best results, as summarized in Table 2. In Table 3 is presented the power output of each thermoelectric Unit of this system.

The power output dispatch of the BBO(Bhattacharya and Chattopadhyay, 2010b), DE/BBO (Bhattacharya and Chattopadhyay, 2010a) and GWO(Zhan et al., 2015a) is slightly better, but the fuel type determined by each mentioned method is the same, showing the efficiency of proposed method. So, for more critical analysis the BA and EBA are evaluated by box plot, being considers as comparative variables the operation cost and the iteration that the good quality solution is found, for each method are performed 50 evaluations. Based on evaluation of methods the more efficients are compared and detailed below, being EBA_0, which represents just modification of analysis to local search, BA and EBA with rate PER of 60% that are called BA_60 and EBA_60, respectively.

Table 2. Comparison of the operation cost - System 1

Method	**Best Operation Cost** (\$/h)
IGA-MU(Chiang, 2005)	624.5
NPSO-LRS(Selvakumar and Thanushkodi, 2007)	624.1
APSO[1](Selvakumar and Thanushkodi, 2008)	624.0
APSO[2](Panigrahi et al., 2008)	623.9
BBO(Bhattacharya and Chattopadhyay, 2010b)	**605.6**
DE/BBO(Bhattacharya and Chattopadhyay, 2010a)	**605.6**
ABC(Hemamalini and Simon, 2010)	609.2
IHS(Arul et al., 2014)	623.8
DSD(Zhan et al., 2015b)	623.8
MPSO(Basu, 2015)	607.9
KGMO(Basu, 2016)	608.1
BSA(Modiri-Delshad et al., 2016)	623.9
IODPSO-L(Qin et al., 2017) .	623.8
KWA-IV(Mandal et al., 2014)	605.7
GWO(Zhan et al., 2015a)	**605.6**
BA	**605.7**
Proposed EBA	**605.7**

Table 3. Results of power output for System 1

Thermoelectric Unit	**Fuel Type**	**Generation (MW)**
	BA and **EBA**	
1	2	212.5735
2	1	212.3825
3	3	332.0000
4	3	239.1304
5	1	268.8913
6	3	239.1304
7	1	279.4107
8	3	239.1304
9	3	411.7903
10	1	265.5604
Power output(MW)		2700.00
Operation cost (\$/h)		**605.7**

Operation cost analysis: In this analysis, based on Table 2, is noted that for all variation of proposed methods and the original BA, the power output and operating cost are equal, differing in the iteration in which the better solution is found, based on the analysis below.

Iteration analysis: To iteration analysis is investigated, for each evaluation, the iteration that the optimal solution is found, the box plot by each method shown in Figure 1 with their data presented by Table 4.

Table 4. Box plot parameters - Iteration - System 1

Methodology	**BA**	**BA_60**	**EBA_0**	**EBA_60**
Minimum	4	4	4	4
1st quartile	4	6	4	5
Median	6.5	9	5	8
3rd quartile	9	12	7	10
Maximum	10	32	9	19
IQR	5	6	3	5
Outliers	-	32	-	19

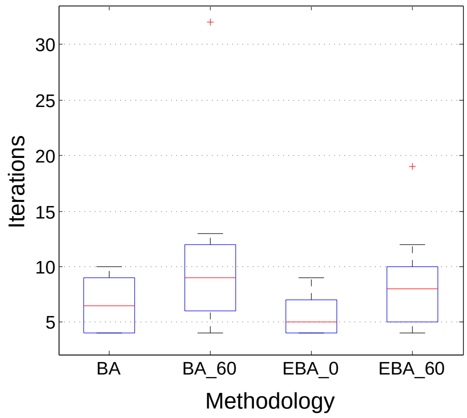

Figure 1. Methodology evaluation - Iteration - System 1

The analysis of these methods by box plot shows the data distributed asymmetrically as illustrated in its respective box plot presented in Figure 1, which represents the skewed data. Other fact consists of the existence of outliers when the rate PER is 60%. Therefore, by the lower median value and the IQR the best method for this case study is considered EBA_0, with 75% of optimal solution being found between 4 to 7 iteration.

Thus, for this study case there is improvement in the convergence process when is used the proposed EBA with only modification in local search strategy, reducing the iteration that the optimal solution is found, that consist on the same iteration number.

4.2 System 2: 40 thermoelectric units with practical constraints

To study all practical constraints described in this paper is creates a new case study based on the Taipower case presented by Chen and Chang (1995) with POZ described in Naresh et al. (2004), with some modifications to insert the valve-point effect and multiple fuels constraints, which has not been considered in the original case. So, this system consists of 40 thermoelectric units, whose presence of practical constraints refer to the multiple fuels, valve-point effect and POZ varies for each unit and a total demand of 7000MW, the ramp limits has not be considered. The complete data of this system is available via following link: `https://github.com/esogit/EDSystem.git`.

How this system is proposed by this paper, this ED problem is solved besides the EBA and BA and also by Trelea-PSO (Oliveira et al., 2015) that has the search process based on social behavior of particles and group (swarm) in the optimization process. The minimum operation cost and the power output of each thermoelectric unit determined by methods mentioned are presented in Table 5.

It is observed that the EBA and BA presented the same optimal solution, but differ from Trelea-PSO. These methods present the same fuel type to thermoelectric units, the difference in total operation cost that consists in allocation of active power and in the determination of operation zone of the units 3, 20, 21 and 26. For example, the thermoelectric unit 26 in BA and EBA methods operates above the upper bound of POZ 2 with dispatch of 520.02MW, while that the Trelea-PSO method operates in the lower bound

Table 5. Results of optimization process for created System 2

Thermoelectric Unit	BA and EBA Fuel Type	BA and EBA Generation (MW)	Trelea-PSO Fuel Type	Trelea-PSO Generation (MW)
1	1	40.0000	1	40.0000
2	-	60.0000	-	60.0000
3	-	97.4316	-	82.0000
4	-	24.0000	-	24.0000
5	-	26.0000	-	26.0000
6	-	85.9441	-	95.5700
7	-	110.0000	-	110.0000
8	1	192.0000	1	192.0000
9	-	298.0201	-	300.0000
10	1	130.0000	1	130.0000
11	1	94.0000	1	94.0000
12	-	94.0000	-	94.0000
13	1	125.0000	1	125.0000
14	2	280.0000	2	294.5601
15	-	125.0000	-	125.0000
16	1	125.0000	1	125.0000
17	-	125.0000	-	125.0000
18	-	327.1900	-	362.2015
19	3	500.0000	3	500.0000
20	-	327.1895	-	362.3349
21	-	327.1887	-	362.3335
22	-	550.0000	-	550.0000
23	3	550.0000	3	550.0000
24	2	550.0000	2	550.0000
25	-	550.0000	-	550.0000
26	-	520.0222	-	380.0000
27	-	526.0138	-	550.0000
28	-	10.0000	-	10.0000
29	1	10.0000	1	10.0000
30	-	10.0000	-	10.0000
31	1	20.0000	1	20.0000
32	-	20.0000	-	20.0000
33	-	20.0000	-	20.0000
34	-	20.0000	-	20.0000
35	1	18.0000	1	18.0000
36	-	18.0000	-	18.0000
37	-	20.0000	-	20.0000
38	-	25.0000	-	25.0000
39	1	25.0000	1	25.0000
40	1	25.0000	1	25.0000
Power output (MW)		7000.00		7000.00
Operation cost ($/h)		**96463.0**		**96539.4**

of POZ 1, with dispatch of 380MW. Therefore, in most complex problems the EBA and BA methods proved to be more powerful. Details of efficiency analysis between BA and EBA based on comparison the minimum operation cost and iteration of optimal solution is found by both methods are presented as following.

Operation cost analysis: The data referring to the minimum operation cost for each evaluation is shown in box plot of Figure 2, with parameters detailed by Table 6.

Through of the box plot analysis from operation cost is presumed that the proposed EBA improves the repeatability of the optimal solution, showing the convergence to the same value in all evaluation. The outliers does not occur to proposed EBA contributing to the efficient of proposed EBA.

Iteration analysis: In this analysis, the Figure 3 presented the box plot of iteration that optimal solution is

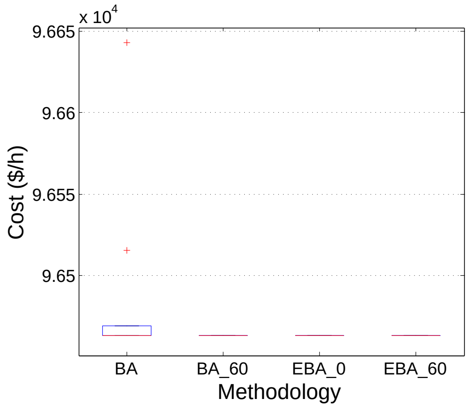

Figure 2. Methodology evaluation - Operation cost - System 2

Table 6. Box plot parameters - Operation cost - System 2

Methodology	**BA**	**BA_60 / EBA_0 / EBA_60**
Minimum	96,463.0	96,463.0
1st quartile	96,463.0	96,463.0
Median	96,463.0	96,463.0
3rd quartile	96,469.1	96,463.0
Maximum	96,643.1	96,463.0
IQR	6.13	0
Outliers	96,515.4	-
	96,643.1	-

found by the use BA, BA_60, EBA_0 and EBA_60 methods. The box plot parameters are presented in Table 7.

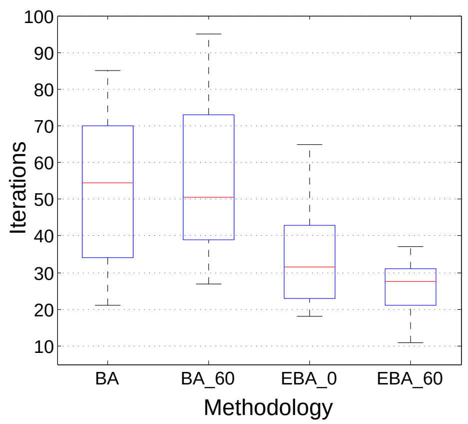

Figure 3. Methodology evaluation - Iteration - System 2

Table 7. Box plot parameters - Iterations - System 2

Methodology	**BA**	**BA_60**	**EBA_0**	**EBA_60**
Minimum	21	27	18	11
1st quartile	34	39	23	21
Median	54.5	50	31.5	27.5
3rd quartile	70	73	43	31
Maximum	85	95	65	37
IQR	36	34	20	10
Outliers	-	-	-	-

In the comparison of the original BA with only one modification at a time in EBA is noted that when the rate PER equal 60% occurs a reduction from 4.5 iterations when comparing the medians and 2 iterations in relation the IQR, other important observation is that randomness increase also increase the variation of iteration to maximum 95, as shown in the top whiskers that represents 25% of the data are between 73 to 95. Already, the change in local search strategy, EBA_0, the reduction is more significant resulting the value 31.5 and 20 for the median and IQR, respectively.

When the modifications are combined results in the EBA_60 that is the better method for this study, having its efficiency proved by lowest value of box plot parameters compared to other methods. The box plot from EBA_60 shown that 25% of all evaluations have the optimal solution obtained between 11 to 21 iterations and none outliers.

The box plots related with iteration and operation cost shown in Figure 3 and Figure 2, respectively, represents the asymmetrical distribution to results of economic dispatch, allowing to evaluate the convergence tendency of each methods, for example, the EBA_60 shows that in 25% of evaluations the optimal solution is found is between the median to third quartiles (27.5 to 31), resulting in the smallest variation of iterations when compared with other methods, as shown in Figure 3.

Among the methods evaluated by statistic analysis the better performance to solve the proposed ED problem, which consists of determining the optimal solution in the smallest iteration, is obtained by proposed EBA with PER equal 60% (EBA_60). As seen in the Figure 3 and Figure 2, the EBA_60 converge to the same operation cost in a minimum variation in your iteration. The Figure 4 represents the convergence process of BA and EBA_60 in the minimum iteration of each.

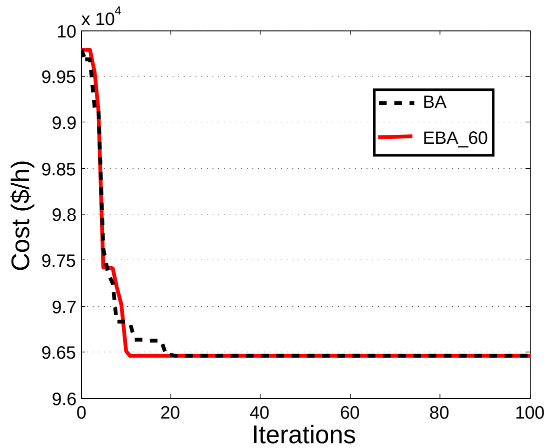

Figure 4. Convergence process evaluation - System 2

Although the BA has presented good performance, the proposed modification impact positively in solved the thermal economic dispatch problem. The Figure 4 makes evident the efficiency of proposed method EBA_60 due to the minimum operation cost to be determined in lower variation of iteration by reason of improved search process associated with rate PER.

Even with the increasing complexity of the problem, the proposed EBA proves to be efficient getting a good quality solution that is found in minimum number of iteration to the studied cases.

5. CONCLUSION

The thermal Economic Dispatch problem studied in this paper can be classified as nonlinear with discontinuities and nonconvexities, which appear due to combination of the practical constraints. So, the solution of case study presented in literature and the system that combines all operations constraints, which describes an unprecedented case study, has been studied through of a recent optimization method, the BA, and the proposed Enhanced Bat Algorithm (EBA).

The research presented in this paper has two main contributions: (*i*) the creation of a new case study, making it the most complex system due to variation in the representation of thermoelectric unit, whose presence of valve-point effect, multiple fuels and POZ varies for each thermoelectric unit; (*ii*) the proposed EBA method to solve the thermal ED problem that is an algorithm simple, easily implementable and robust that can find accurate, consistent and good quality solution. The proposed EBA holds similarities with BA, but its modifications make it a method more efficient to solve the thermal ED problem, it is proving to be competitive with other from literature as in consolidated study case and the newly created system, presenting best solution or of equal quality and the capacity of keep the repeatability of the optimal solution that is found in a minimum number of iterations.

Based on the results obtained in the whole set of case studies and comparisons with results from the technical literature, this paper demonstrates the effectiveness of the proposed EBA method through statistic analysis and this technique proves promising to be applied to solve thermal ED problems, including large scale systems where the EBA characteristic is evidenced by best exploration of the feasible space.

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