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Reference Model-based Fault Detection and Isolation for Discrete-time Systems Subject to Persistent Disturbances ⋆

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Abstract: This paper deals with the design of a robust filter aimed for fault detection and isolation applied to discrete-time systems subject to arbitrary (not necessarily vanishing) norm-bounded (i.e., ℓ_∞) input disturbances. The idea is to approximate the behavior from faults to residual given by a reference model despite the presence of disturbances. The filter design is cast as an optimization problem subject to linear matrix inequality constraints. A numerical example is presented to demonstrate the potential of the proposed approach.

Keywords: Peak norm; H_- index; Fault detection and isolation; Reference model; Linear matrix inequalities.

1. INTRODUCTION

In recent years, model-based fault detection and isolation (FDI) has attracted more attention as a result of the increasing demand for safety and reliability of dynamic systems (Frank and Ding, 1997). In order to obtain a reliable FDI, the sensitivity to faults and the robustness against exogenous inputs should be considered (Ding et al., 2000). The focus of this work relies on ensuring fault detection and isolation for linear discrete-time systems subject to disturbances and faults (in actuators and sensors). The main idea is to generate residuals in order to obtain a reasonable trade-off between sensitivity to faults and insensitivity to disturbances. In this sense, a norm-based approach has proven to be efficient for residual evaluation purposes (Khan et al., 2010). For instance, the H_- index is normally used to characterize the worst-case sensitivity of the residual to the fault, as it is presented by Henry et al. (2014) and Li and Liu (2013). Solutions to the FDI problem based on a multi-objective approach have been developed in numerous works. For instance, Wang et al. (2007) uses the H_- index for fault sensitivity evaluation as well as the worst-case robustness measure, the H_∞ norm, to design an observer aimed for fault detection. Similar approaches are presented by Li and Liu (2013) and Liu et al. (2003). Additionally, LMI-based methods have been widely studied (Zhong et al., 2003), (Casavola et al., 2005). Even though the H_∞ norm is widely applied for FDI purposes, the so-called Peak norm has achieved promising results. While the H_∞ norm takes into account the energy gain of a system, the Peak norm considers its peak gain. This can be an asset when one seek to ensure the boundedness of signals such as persistent disturbances. The present work follows this route, where the Peak norm is studied for evaluating the disturbance effect on the residual signal.

In the above line of research, the focus is on robust fault detection. The problem of enforcing fault isolation has been addressed by several authors, e.g., Patton and Chen (1997), Stoustrup and Niemann (2002). Following this route, a series of multi-objective FDI problems has been investigated by using a given reference model describing the desired response. For instance, Frisk and Nielsen (2006) study the design of an H_∞ residual generator in order to enforce a desired behavior for the residual response with respect to the faults and taking into account model uncertainties. Similarly, Nobrega et al. (2008) presents the design of LMI-based H_∞ filters considering a reference model that characterizes the response of the residual to the faults in order to guarantee fault detection and isolation despite the presence of disturbances and model uncertainties.

Therefore, as this work is motivated by the guarantee of fault detection and isolation for linear discrete-time systems, a robust filter is designed taking into account the H_- index as a fault sensitivity measure as well as the Peak norm for disturbance effect evaluation. One considers a triangular reference model structure with partially fixed dynamics. Such a structure allows one to consider a situation where more faults than the number of measured system outputs are present. This notably occurs when all sensors and actuators can be subject to faults. The focus is on the design of the residual generator including a part of

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its structure. The problem is cast as a convex optimization problem subject to LMI constraints.

The paper is organized as follows. The problem statement is presented in Section 2. The filter design for fault detection and isolation purposes is addressed in Section 3. Finally, a numerical example is given in Section 4 to illustrate the effectiveness of the approach.

Notation. $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of $m\times n$ real matrices, and I_n is the $n\times n$ identity matrix. For a real matrix S, S^T denotes its transpose, and $S>0$ (≥ 0) means that S is symmetric and positive-definite (positive semi-definite). For a symmetric block matrix, $*$ stands for the transpose of the blocks outside the main diagonal block. For a transfer function, z defines its representation on the $\mathcal{Z}$-domain. The one-step-ahead shift operator of a sequence $f(k)$ is denoted as $f^+ := f(k+1)$. For a matrix transfer function $\mathcal{G}$, $\|\mathcal{G}\|_{peak}$ and $\|\mathcal{G}\|_{-}$ represent its peak value and its smallest singular value, respectively.

2. PROBLEM STATEMENT

Consider the following linear discrete-time system:

$$\begin{cases} x^+ = Ax + B_u u + B_w w + B_f f, \\ y \;\;= Cx + D_u u + D_w w + D_f f, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$ are the measured input and output, respectively, $w \in \mathcal{W} \subset \mathbb{R}^{n_w}$ is an exogenous input, $f \in \mathcal{F} \subset \mathbb{R}^m$ is a vector containing actuator and/or sensor faults, and $A, B_u, B_w, B_f, C, D_u, D_w, D_f$ are given real matrices with appropriate dimensions. Moreover, assume the following conditions with respect to the system (1):

A1 $n_u = n_y = n$.

A2 $m \leq 2n$.

A3 $\mathcal{W} = \{w \in \mathbb{R}^{n_w} : w^T w \leq 1\}$.

A4 $\mathcal{F} = \{f \in \mathbb{R}^m : f^T f \leq 1\}$.

Notice for magnitude bounded signals that assumptions (A3) and (A4) can be made without loss of generality by properly rescaling the matrices B_w, D_w, B_f and D_f.

Thus, taking into account the system in (1), the following observer-like filter is proposed:

$$\begin{cases} \hat{x}^+ = A\hat{x} + B_u u + L(y - \hat{y}), \\ \hat{y} \;\;= C\hat{x} + D_u u, \\ r \;\;= QC_r(y - \hat{y}), \end{cases} \tag{2}$$

where $\hat{x} \in \mathbb{R}^{n_x}$ and $\hat{y} \in \mathbb{R}^{n_y}$ are the estimation of the state and output vectors, respectively, $r \in \mathbb{R}^m$ represents the residual vector, $C_r \in \mathbb{R}^{m\times n}$ is a given matrix, and the triangular matrix $Q \in \mathbb{R}^{m\times m}$ and the observer matrix gain $L \in \mathbb{R}^{n_x\times n_y}$ are to be designed. Next, considering the estimation error vector

$$\tilde{x} := x - \hat{x}, \tag{3}$$

the following alternative expression for the residual is obtained:

$$\begin{cases} \tilde{x}^+ = (A - LC)\tilde{x} + (B_w - LD_w)w + (B_f - LD_f)f, \\ r \;\;= QC_r(y - \hat{y}). \end{cases} \tag{4}$$

In order to ensure some performance with respect to fault sensitivity, consider the following reference model regarding the behavior from faults to the residual:

$$\begin{cases} \breve{x}^+ = \breve{A}\breve{x} + \breve{B}f, \\ \breve{r} \;\;= Q(\breve{C}\breve{x} + \breve{D}f), \end{cases} \tag{5}$$

where $\breve{x} \in \mathbb{R}^q$ is the reference model state, $\breve{r} \in \mathbb{R}^m$ is the reference model residual, and $\breve{A}$, $\breve{B}$, $\breve{C}$ and $\breve{D}$ are given matrices with appropriate dimensions such that the transfer function

$$\breve{\mathcal{G}} = \breve{C}(zI - \breve{A})^{-1}\breve{B} + \breve{D}$$

has an appropriate structure for FDI purposes, with $\breve{A}$ being Schur stable.

Therefore, considering the system model in (1), the main idea is to design a robust filter (4) such that the behavior from faults to residual is as close as possible to the behavior given by the reference model (5). The residual generator design problem is illustrated in Fig. 1, where $e_r = r - \breve{r}$ represents the residual error with respect to the reference model.

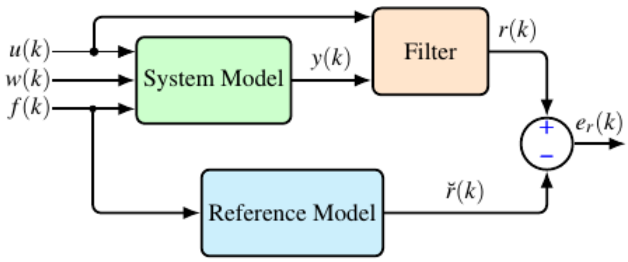


Figure 1. Block diagram representation for FDI purposes.

Hence, the problem of interest in this paper consists in determining the matrices L and Q of the filter in (4) such that:

I) $\|\mathcal{G}_{wr}\|^2_{peak} \leq \gamma_w$,

II) $\|\mathcal{G}_{fr} - \mathcal{G}_{f\breve{r}}\|^2_{peak} \leq \gamma_f$,

III) $\|\mathcal{G}_{f\breve{r}}\|^2_{-} \geq \gamma_c$,

where $\mathcal{G}_{wr}$, $\mathcal{G}_{fr}$ and $\mathcal{G}_{f\breve{r}}$ represent the transfer functions from disturbance to residual, from fault to residual and from fault to reference residual, respectively, and γ_c, γ_w and γ_f are positive scalars defining the residual generator performance.

3. FILTER DESIGN

In order to derive a solution to the filter design problem, let

$$V_1(\tilde{x}) = \tilde{x}^T P_1 \tilde{x},\; P_1 \in \mathbb{R}^{n_x\times n_y},\; P_1 > 0,$$

be a Lyapunov function candidate for the estimation error in (4), and consider the following inequality:

$$\Delta V_1(\tilde{x}) \leq \tau_1(w^T w - V_1(\tilde{x})), \quad \tau_1 \in (0,1), \tag{6}$$

where $\Delta V_1(\tilde{x}) = V_1(\tilde{x}^+) - V_1(\tilde{x})$. If the condition in (6) is satisfied, then the estimation error system in (4) is input-to-sate (ISS) stable (Sontag and Wang, 1996). Moreover, it can be shown that the following matrix inequality is a sufficient condition for ensuring that (6) holds for some $\tau_1 \in (0,1)$:

$$\begin{bmatrix} P_1 - K^T - K & KA - L_k C & B_w - L_k Dw \\ * & -(1-\tau_1)P_1 & 0 \\ * & * & -\tau_1 I_{n_w} \end{bmatrix} < 0, \quad (7)$$

where $K \in \mathbb{R}^{n_x \times n_x}$ is a nonsingular matrix and $L_k = KL$.

In addition, to provide a bound γ_w on $\|\mathcal{G}_{wr}\|_{peak}$, the following must be guaranteed:

$$\gamma_w - r^T r \geq 0, \ \forall\, (\tilde{x}, w) : \tilde{x}^T P_1 \tilde{x} \leq 1, \quad w^T w \leq 1. \quad (8)$$

Furthermore, if there exists positive scalars α_1 and β_1 such that

$$\alpha_1 w^T w + \beta_1 \tilde{x}^T P_1 \tilde{x} - r^T r \geq 0 \quad (9)$$

is satisfied, then (8) holds with $\gamma_w = \alpha_1 + \beta_1$. In order to obtain a tractable solution, (9) can be cast as

$$\begin{bmatrix} \tilde{x} \\ w \end{bmatrix}^T \left(\begin{bmatrix} P_1 & 0 \\ 0 & \rho_1 I_{n_w} \end{bmatrix} + \eta_1 \begin{bmatrix} C^T C_r^T \\ D_w^T C_r^T \end{bmatrix} Q^T Q \begin{bmatrix} C^T C_r^T \\ D_w^T C_r^T \end{bmatrix}^T \eta_1 \right) \begin{bmatrix} \tilde{x} \\ w \end{bmatrix} \geq 0 \quad (10)$$

where $\rho_1 = \alpha_1/\beta_1$ and $\eta_1 = 1/\sqrt{\beta_1}$, with $\gamma_w = (\rho_1 + 1)\eta_1^{-2}$.

Then, by applying the Schur's complement, the following LMI is a necessary and sufficient condition for (10):

$$\begin{bmatrix} P_1 & * & * \\ 0 & \rho_1 I_{n_w} & * \\ \eta_1 C_r C & \eta_1 C_r D_w & Q_r \end{bmatrix} > 0, \quad (11)$$

where $Q_r = Q^{-1} Q^{-T}$.

Next, taking into account the residual generator in (4) and the reference model in (5), the following augmented system can be defined

$$\begin{cases} \bar{x}^+ = \bar{A}\bar{x} + \bar{B}f, \\ e_r = Q(\bar{C}\bar{x} + \bar{D}f), \end{cases} \quad (12)$$

where $\bar{x} = \begin{bmatrix} \tilde{x}^T & \breve{x}^T \end{bmatrix}^T$ and

$$\bar{A} = \begin{bmatrix} A - LC & 0 \\ 0 & \breve{A} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_f - LD_f \\ \breve{B} \end{bmatrix}, \quad (13)$$
$$\bar{C} = \begin{bmatrix} C_r C & -\breve{C} \end{bmatrix}, \quad \bar{D} = C_r D_f - \breve{D}.$$

Thus, in order to determine a bound on

$$\|\mathcal{G}_{fe_r}\|_{peak} := \|\mathcal{G}_{fr} - \mathcal{G}_{f\breve{r}}\|_{peak}, \quad (14)$$

let $V_2(\bar{x}) = \bar{x}^T P_2 \bar{x}$, $P_2 \in \mathbb{R}^{n_a \times n_a}$, $n_a := n_x + q$, $P_2 > 0$, and consider:

$$\Delta V_2(\bar{x}) \leq \tau_2(f^T f - V_2(\bar{x})), \quad \tau_2 \in (0,1), \quad (15)$$

with $\Delta V_2(\bar{x}) = V_2(\bar{x}^+) - V_2(\bar{x})$. Then, considering

$$P_2 = \begin{bmatrix} P_{21} & P_{22} \\ P_{22}^T & P_{23} \end{bmatrix}, \quad \bar{K} = \begin{bmatrix} K & K_3 \\ MK & K_4 \end{bmatrix}, \quad (16)$$

with K, $K_3 \in \mathbb{R}^{n_x \times n_q}$ and $K_3 \in \mathbb{R}^{n_q \times n_q}$ to be designed and $M \in \mathbb{R}^{n_q \times n_x}$ be given, we obtain:

$$\begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \omega_{14} & \omega_{15} \\ * & \omega_{22} & \omega_{23} & \omega_{24} & \omega_{25} \\ * & * & \omega_{33} & \omega_{34} & 0 \\ * & * & * & \omega_{44} & 0 \\ * & * & * & * & \omega_{55} \end{bmatrix} > 0, \quad (17)$$

where

$$\omega_{11} = P_{21} - K - K^T, \ \omega_{12} = P_{22} - K_3 - K^T M^T,$$
$$\omega_{13} = KA - L_k C, \ \omega_{14} = K_3 \breve{A}, \ \omega_{24} = K_4 \breve{A},$$
$$\omega_{15} = KB_f - L_k D_f + K_3 \breve{B}, \ \omega_{33} = -(1-\tau_2)P_{21},$$
$$\omega_{22} = P_{23} - K_4 - K_4^T, \ \omega_{23} = MKA - ML_k C,$$
$$\omega_{25} = MKB_f - ML_k D_f + K_4 \breve{B}, \ \omega_{55} = -\tau_2 I_m,$$
$$\omega_{34} = -(1-\tau_2)P_{22}, \ \omega_{44} = -(1-\tau_2)P_{23}.$$

Hence, a bound γ_f on $\|\mathcal{G}_{fe_r}\|_{peak}$ is guaranteed if the following holds:

$$\gamma_f - e_r^T e_r \geq 0, \ \ \forall\, (\bar{x}, f) : \bar{x}^T P_2 \bar{x} \leq 1, \quad f^T f \leq 1. \quad (18)$$

Similarly to the computation of a bound on $\|\mathcal{G}_{wr}\|_{peak}$, if the following matrix inequality is satisfied

$$\begin{bmatrix} P_{21} & * & * & * \\ P_{22}^T & P_{23} & * & * \\ 0 & 0 & \rho_2 I_m & * \\ \eta_2 C_r C & -\eta_2 \breve{C} & \eta_2 C_r D_f - \eta_2 \breve{D} & Q_r \end{bmatrix} > 0, \quad (19)$$

then $\|\mathcal{G}_{fe_r}\|_{peak} \leq \gamma_f$, with $\gamma_f = (\rho_2 + 1)\eta_2^{-2}$.

Next, from Li and Liu (2013), the condition III is satisfied if and only if the following holds:

$$\begin{bmatrix} \omega_{r_{11}} & * \\ \omega_{r_{21}} & \omega_{r_{22}} \end{bmatrix} \geq 0, \quad (20)$$

where $P_3 = P_3^T \in \mathbb{R}^{q \times q}$ is to be designed, and $\omega_{r_{11}} = \breve{A} P_3 \breve{A}^T - P_3 + \breve{B}\breve{B}^T$, $\omega_{r_{21}} = \breve{C} P_3 \breve{A}^T + \breve{D}\breve{B}^T$ and $\omega_{r_{22}} = \breve{C} P_3 \breve{C}^T + \breve{D}\breve{D}^T - \gamma_c Q_r$.

Thus, the following LMI-based theorem is established with a view to ensure that the residual generator performance specifications I to III hold.

Theorem 1. Consider the residual generator in (4), the reference model in (5) and the augmented system in (12). Let $\breve{A}$, $\breve{B}$, $\breve{C}$, $\breve{D}$, M, C_r, γ_c, τ_1 and τ_2 be given, with $\tau_i \in (0,1)$, $i = 1, 2$. Suppose there exist symmetric matrices P_1, P_{21}, P_{23}, P_3 and Q_r, free matrices L_k, K, K_3, K_4, and P_{22}, and positive scalars ρ_1, ρ_2, η_1 and η_2 such that the LMIs in (7), (11), (17), (19) and (20) are satisfied. Then, the following statements hold:

i) the estimation error system (4) is ISS;

ii) $\|\mathcal{G}_{wr}\|_{peak} \leq \frac{\sqrt{1+\rho_1}}{\eta_1}$;

iii) $\|\mathcal{G}_{fe_r}\|_{peak} \leq \frac{\sqrt{1+\rho_2}}{\eta_2}$; and

iv) $\|\mathcal{G}_{f\breve{r}}\|_{-} \geq \gamma_c$,

with $L = K^{-1} L_k$ and $Q = Q_r^{-1/2}$.

Proof: See the Appendix.

Thus, an optimized residual generator is derived by means of the solution of the following optimization problem:

$$\min_{P_1, \ldots, Q_r, \gamma_w} \gamma_f \text{ subject to } \begin{cases} (7), (11), (17), \\ (19), (20). \end{cases} \quad (21)$$

4. NUMERICAL EXAMPLE

Consider the system in Fig. 2, which is composed by three identical cylindrical tanks with transverse area S, linked to each other by cylindrical tubes with transverse area

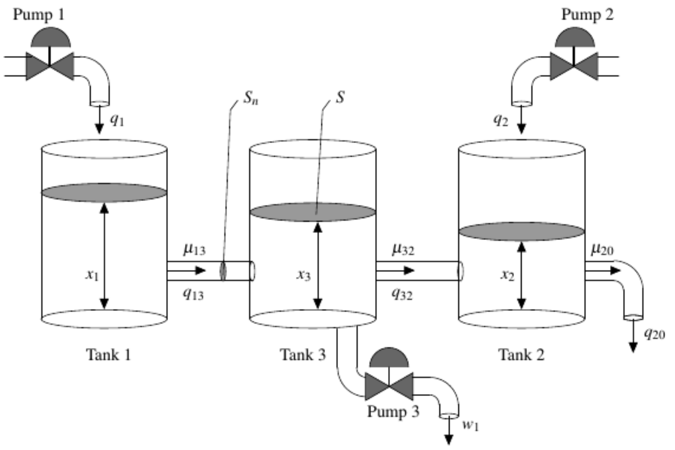


Figure 2. Three tanks system.

S_n and equal flow coefficients μ_{13} and μ_{32}. The output pipe in Tank 2 has the same tranverse area S_n, but a different flow coefficient μ_{20}. The actuator valves (Valve 1 and Valve 2), which flow rates are represented by q_1 and q_2, respectively, are in charge of filling Tanks 1 and 2. For physical reasons, the tank levels x_i (for $i = 1, 2, 3$) and the valve flow rates q_j (for $j = 1, 2$) are bounded by $x_{i_{max}}$ (for $i = 1, 2, 3$) and $q_{j_{max}}$ (for $j = 1, 2$), respectively. Additionally in this work, we consider a disturbance input represented by $w = \begin{bmatrix} w_1^T & w_2^T \end{bmatrix}^T$, where w_1 is the flow of the pump and w_2 is an energy bounded noise on the level sensor of Tank 1. Additive faults on Valve 1 and on the measuring sensors of Tanks 1 and 3 are considered.

Consider the following numerical parameters for the system:

$$\begin{aligned} &S = 0.0154 \text{ m}^2, \quad S_n = 5 \times 10^{-5} \text{ m}^2, \\ &x_{i_{max}} = 0.62 \text{ m}, \quad q_{j_{max}} = 1.2 \times 10^{-4} \text{ m}^3\text{s}^{-1}, \\ &\mu_{13} = \mu_{32} = 0.5, \quad \mu_{20} = 0.675. \end{aligned} \tag{22}$$

The inputs q_1 and q_2 are normalized to belong to the range $q_i \in [0, 1]$, for $i = 1, 2$. Thus, the operating conditions for the inputs are

$$q_1 = 0.2916, \quad q_2 = 0.5417, \tag{23}$$

and the state equilibrium points are equal to

$$x_{s_1} = 0.6115, \; x_{s_2} = 0.4252, \; x_{s_3} = 0.5118, \tag{24}$$

The linear approximate model of the three tanks system can be written as in (1), with

$$\begin{aligned} &A = \begin{bmatrix} 0.9670 & 0.0006 & 0.0324 \\ 0.0006 & 0.9433 & 0.0344 \\ 0.0324 & 0.0344 & 0.9328 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ &B_u = 10^{-3} \begin{bmatrix} 22.9864 & 0.0047 \\ 0.0047 & 22.7053 \\ 0.3855 & 0.4106 \end{bmatrix}, \; D_u = 0_{2\times 3}, \\ &B_f = 10^{-3} \begin{bmatrix} 22.9864 & 0 & 0 \\ 0.00047 & 0 & 0 \\ 0.3855 & 0 & 0 \end{bmatrix}, \; D_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ &B_w = 10^{-3} \begin{bmatrix} \text{-}0.0161 & 0 \\ \text{-}0.0171 & 0 \\ \text{-}0.9407 & 0 \end{bmatrix}, \; D_w = \begin{bmatrix} 0 & 0.05 \\ 0 & 0 \end{bmatrix}, \end{aligned} \tag{25}$$

which leads to the following transfer function from faults to output y:

$$\mathcal{G}_{sys} = \begin{bmatrix} g_{11} & 1 & 0 \\ g_{21} & 0 & 1 \end{bmatrix}, \tag{26}$$

where

$$\begin{aligned} g_{11} &= \frac{0.02299z^2 \text{ - } 0.04311z + 0.02019}{z^3 \text{ - } 2.843z^2 \text{ - } 2.692z - 0.8488}, \\ g_{21} &= \frac{0.0003855z^2 + 8.835 \cdot 10^{-6}z \text{ - } 0.0003509}{z^3 \text{ - } 2.843z^2 + 2.692z \text{ - } 0.8488}. \end{aligned}$$

The system static gain from faults to measurements is given by:

$$K_{sys} = \begin{bmatrix} 2.369 & 1 & 0 \\ 1.684 & 0 & 1 \end{bmatrix}. \tag{27}$$

Notice that the dynamics given by (26) has an appropriate structure represented by the static gain in (27) that ensures fault detection and isolation (as explained later in this section). Therefore, the reference model in (5) is chosen such that the structure given by (27) is kept:

$$\breve{A} = A, \; \breve{B} = B_f, \; \breve{C} = \begin{bmatrix} C \\ 0_{1\times 3} \end{bmatrix}, \; \breve{D} = \begin{bmatrix} D_f \\ 0_{1\times 3} \end{bmatrix},$$

where the third line of $\breve{C}$ and $\breve{D}$ is included to ensure the same number of residuals and faults. The static gain of the reference model is equal to:

$$K_{ref} = \begin{bmatrix} K_{sys} \\ 0_{1\times 3} \end{bmatrix}. \tag{28}$$

From K_{sys}, notice that only two residuals are sufficient to guarantee fault isolation provided that the faults do not occur simultaneously. This means that one can consider only the first two significant rows of the static gain matrix for FDI purposes. Thus, a structure regarding the behavior from faults to residual is enforced by the reference static gain (28) such that:

- both residuals r_1 and r_2 are influenced by the occurrence of an actuator fault (f_a);
- only residual r_1 is affected in case a fault occurs in sensor 1 (f_{s_1});
- similarly, only residual r_2 is influenced by a fault in sensor 2 (f_{s_2}).

Firstly, in this example, we define:

$$\begin{aligned} &\gamma_c = 0.125, \; \tau_1 = 0.001, \; \tau_2 = 0.006, \\ &M = \text{-}0.9 \cdot I_{(n_q \times n_x)}, \; C_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T. \end{aligned} \tag{29}$$

The above parameters are defined such that the LMI conditions are feasible. In particular, a gridding technique was applied for designing τ_1 and τ_2 with $(\tau_1, \tau_2) \in (0, 1) \times (0, 1)$. The matrices C_r and M were set to be $C_r = \begin{bmatrix} I_2 & 0_{2\times 1} \end{bmatrix}^T$ and $M = \xi \cdot I_{n_q \times n_x}$, with ξ being defined for feasibility purposes, and γ_c was iteratively maximized. The optimization problem (21) leads to:

$$\begin{aligned} &Q = \begin{bmatrix} 0.125 & 0 & 0 \\ \text{-}0.0002 & 0.1252 & 0 \\ 0 & 0 & 182.393 \end{bmatrix}, \\ &L = 10^{-3} \begin{bmatrix} 0.7658 & 0.2161 \\ 0.0248 & \text{-}0.0216 \\ 0.1247 & \text{-}0.1295 \end{bmatrix}, \rho_1 = 0.05, \rho_2 = 0.0027, \\ &\eta_1 = 24.77, \; \eta_2 = 20.4, \; \gamma_w = 0.0414, \; \gamma_f = 0.0024. \end{aligned}$$

The derived static gain with respect to the behavior from faults to residual is

$$K_{fr} = \begin{bmatrix} 0.2702 & 0.1150 & -0.0013 \\ 0.1918 & -0.0077 & 0.1246 \end{bmatrix}, \qquad (30)$$

which validates that fault detection and isolation are guaranteed and, consequently, the design conditions I, II and III are satisfied.

Next, a simulation has been carried out in order to evaluate the performance of the designed residual generator with respect to the FDI problem. Thus, for simulation purposes, we consider control variation $\Delta q_1 = \Delta q_2 = 0.05$ and disturbance $w_1 = 0.01 \sin(10^{-5} + \pi/2)$, with w_2 being a white noise with power equal to 0.0001, a mean of 0 and a variance of 1. Considering an amount of 10% of the operating conditions for the inputs in (23) as well as the state equilibrium points in (24), the faults are defined as follows:

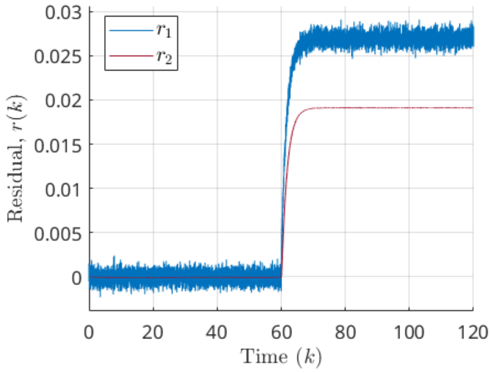
$$f_a = 0.1, \ f_{s_1} = 0.06115, \ f_{s_2} = 0.05118. \qquad (31)$$

Fig. 3 presents the residual response to the application of the control, disturbance and fault inputs. Fig. 3a shows the result related to the application of the actuator fault at instant $k = 60$ (with f_{s_1}=f_{s_2}=0). Furthermore, Fig. 3b and Fig. 3c display the residual signal considering sensor faults f_{s_1} (at instant $k = 70$ with $f_{s2} = 0$ and $f_a = 0$) and f_{s_2} (at instant $k = 80$ with $f_{s1} = 0$ and $f_a = 0$), respectively.

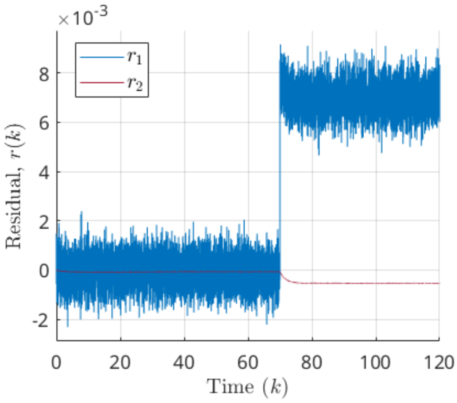
Notice in Fig. 3a that both residuals were affected by the actuator fault f_a as expected. In Fig. 3b, the sensor fault f_{s_1} isolation is guaranteed taking into account the residual r_1 response compared to the small influence on residual r_2. Similarly, in Fig. 3c, the residual r_2 response is sufficiently large to indicate the sensor fault f_{s_2} despite a noisy measurement. From these results, it should be noted the excellent performance achieved by the proposed residual generator (an observer-like filter).

5. CONCLUDING REMARKS

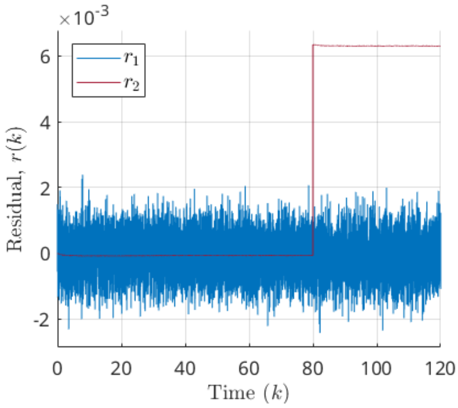
A residual generator based on a robust observer-like filter has been derived for a class of discrete-time systems, where the behavior from faults to residual approximates a given reference model with the view of achieving fault detection and isolation. A convex optimization problem subject to LMI constraints is proposed to synthesize the filter parameters while ensuring the input-to-state stability of the estimation error dynamics. A numerical example was considered to show the effectiveness of the proposed approach.



(a) Residual response to the actuator fault.



(b) Residual response to sensor fault 1.



(c) Residual response to sensor fault 2.

Figure 3. Residual response to fault, control and disturbance inputs.

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Appendix A. PROOF OF THEOREM 1

Firstly, consider the LMI in (7). From the block $(1,1)$, we obtain that $K + K^T - P_1 > 0$. Since $P_1 > 0$, it follows that K is full rank. In addition, notice from

$$(K - P_1)P_1^{-1}(K - P_1)^T \geq 0$$

that the following holds:

$$P_1 - K - K^T \geq -KP_1^{-1}K^T, \tag{A.1}$$

for any nonsingular matrix $K \in \mathbb{R}^{n_x \times n_x}$. Hence, the following inequality holds:

$$\begin{bmatrix} -KP_1^{-1}K & KA - L_kC & B_w - L_kDw \\ * & -(1-\tau_1)P_1 & 0 \\ * & * & -\tau_1 I_{n_w} \end{bmatrix} < 0. \tag{A.2}$$

Next, pre- and post-multiplying the above by

$$\mathrm{diag}\{P_1K^{-1}, I_{n_x}, I_{n_w}\}$$

and its transpose, respectively, and then applying the Schur's complement leads to:

$$\begin{bmatrix} -P_1 & KA - L_kC \\ * & -(1-\tau_1)P_1 \end{bmatrix} - \begin{bmatrix} (B_w - L_kD_w)^T \\ 0 \end{bmatrix} \tau_1 I_{nw} \left[(B_w - L_kD_w) \ 0\right] < 0. \tag{A.3}$$

Hence, pre- and post-multiplying (A.3) by $[\tilde{x}^{\,T} \ \ w^T]^T$ and its transpose yields (6) and thus $\|\mathcal{G}_{wr}\|_{peak} \leq \frac{\sqrt{1+\rho_1}}{\eta_1}$.

Now, suppose that the LMI in (17) is satisfied. Then, the following condition can be obtained:

$$\Omega < 0, \quad \Omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ * & \omega_{22} \end{bmatrix}, \tag{A.4}$$

or equivalently $K_2 + K_2^T - P_2 > 0$, where K_2 and P_2 are as defined in (16). Hence, from the fact that $P_2 > 0$, it follows that K_2 is full rank. Accounting for the fact that

$$P_2 - K_2 - K_2^T \geq -K_2P_2^{-1}K_2^T,$$

it turns out that the LMI in (17), with the block Ω as defined in (A.4) being replaced by

$$-K_2P_2^{-1}K_2^T,$$

holds. Thus, pre- and post-multiplying the resulting matrix inequality by $\mathrm{diag}\{P_2K_2^{-1}, I_{n_a}, I_m\}$ and its transpose, respectively, yields

$$\begin{bmatrix} -P_2 & 0 \\ 0 & -(1-\tau_2)P_{21} \end{bmatrix} + \begin{bmatrix} \bar{A}^T \\ \bar{B}^T \end{bmatrix} P_2 \left[\bar{A} \ \bar{B}\right] - \begin{bmatrix} \bar{C}^T \\ \bar{D}^T \end{bmatrix} \tau_2 I_m \left[\bar{C} \ \bar{D}\right] < 0 \tag{A.5}$$

from the Schur's complement. Further, pre- and post-multiplying (A.5) by $[\bar{x}^T \ \ f^T]^T$ and its transpose, respectively, leads to (15). Hence, it follows that $\|\mathcal{G}_{fe_r}\|_{peak} \leq \frac{\sqrt{1+\rho_2}}{\eta_2}$.

Finally, $\|\mathcal{G}_{f\check{r}}\|_- \geq \gamma_c$ follows straightforwardly from (19) and Li and Liu (2013), which completes the proof. □